Two types of waves

- electromagnetic
- electrostatic

Represented by an osci fu^n

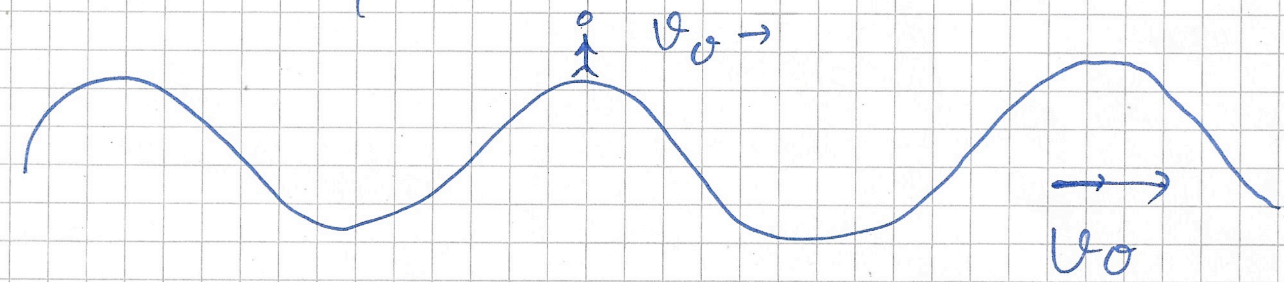
$$\bar{E} = \bar{E}_0 \exp\left[i\,(kx - \omega t)\right]$$

wavenumber $2\pi/\lambda$ $\qquad \omega = 2\pi f$

$$\bar{E} = \bar{E}_0 \cos(kx - \omega t) + i\,\bar{E}_0 \sin(kx - \omega t)$$

phase velocity.

$$\bar{E} = \bar{E} \exp(i\theta)$$



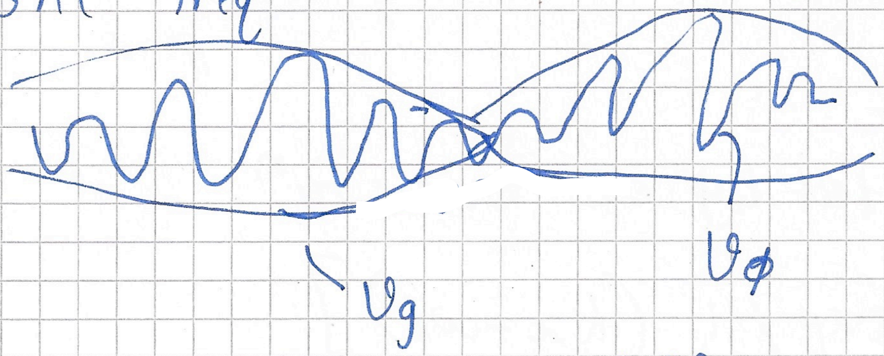
with the crest of wave

$$\frac{d\theta}{dt} = 0 = \frac{d}{dt}(kx - \omega t) = k\underbrace{\frac{dx}{dt}}_{v_\theta} - \omega\frac{dt}{dt}$$

$\Rightarrow$

$$v_\theta = \frac{\omega}{k} \quad \text{phase velocity.}$$

superposition of waves of more than one freq^n



$$v_g \neq v_\phi$$

group velocity

$$v_g = \frac{d\omega}{dk}$$

dispersion Relation

Vlasova eq^n in One-D $\quad \bar{B}=0$, $\bar{E_0}=0$

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = 0$$

when wave is propogating through the electron gas the local distribution fu^n in the absence of wave is Maxwellian $f_0(x,v,t)$ will be perturbed $\tilde{f}(x,v,t)$, new distribution fu^n

$f = f_0 + \tilde{f}$ $\quad$ let $E = \tilde{E}$ small field due to the wave in plasma $\quad E = \bar{E_0} + \tilde{E}$

$$f(\bar{x},\bar{u},t) = f_0(\bar{v}) + \tilde{f}(x,v,t)$$

$$\frac{\partial \tilde{f}}{\partial t} + v\frac{\partial \tilde{f}}{\partial x} + \frac{q}{m}\tilde{E}\frac{\partial f_0}{\partial v} + \underbrace{\frac{q}{m}\tilde{E}\cdot\frac{\partial \tilde{f}}{\partial x}}_{\substack{\text{non linear term} \\ =0}} = 0$$

Linearized analysis.

$$\frac{\partial \tilde{f}}{\partial t} + v\frac{\partial \tilde{f}}{\partial x} + \frac{q\tilde{E}}{m}\frac{\partial f_0}{\partial v} = 0$$

we assume that the perturbation are due to plane waves of form $\exp(ikx-\omega t)$

Then we can put

$$\partial/\partial t \to -i\omega \quad , \quad \frac{\partial}{\partial x} \to ik$$

$$-i\omega\tilde{f} + ikv\tilde{f} + \frac{q\tilde{E}}{m}\frac{\partial f_0}{\partial v} = 0$$

$$(-i\omega + ikv)\hat{f} = \frac{-q\tilde{E}}{m}\frac{\partial f_0}{\partial v}$$

$$\tilde{f} = -\frac{q\tilde{E}}{m}\frac{\partial f_0/\partial v}{i(kv-\omega)}$$

density perturbation can be calculated

$$\tilde{n} = \int_{-\infty}^{\infty} \tilde{f}\, dv$$

$$\therefore \tilde{n} = -\frac{q\tilde{E}}{im}\int_{-\infty}^{\infty} \frac{\partial f_0/\partial v}{(kv-\omega)}\, dv$$

$$\tilde{E} = -\frac{\partial\tilde{\phi}}{\partial x} = -ik\tilde{\phi}$$

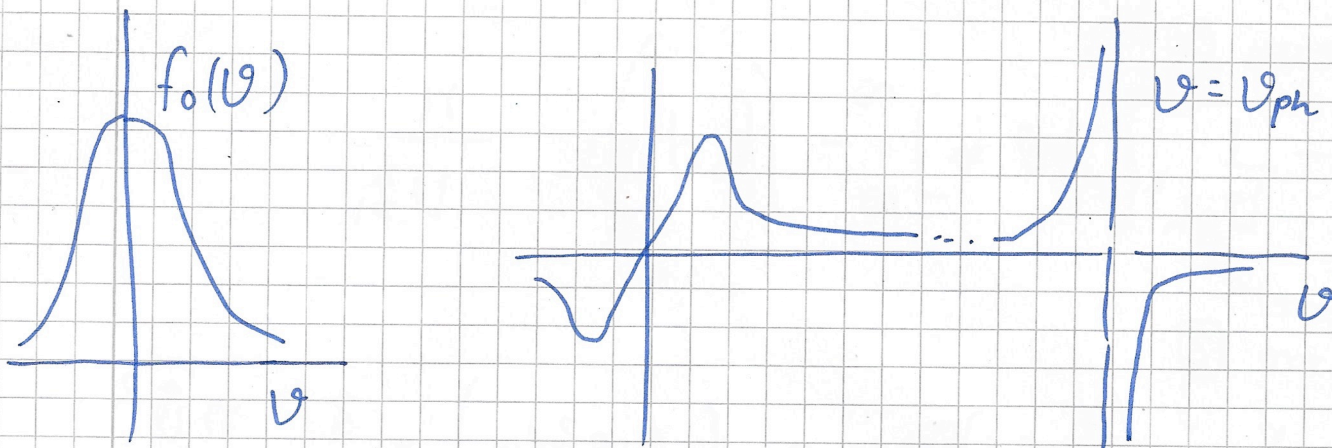
$$\frac{\partial \tilde{E}}{\partial x} = -\rho/\epsilon_0$$

$$\Downarrow \quad k^2\tilde{\phi} = \frac{\tilde{n}q}{\epsilon_0} \Rightarrow \tilde{n} = \frac{k^2\epsilon_0\tilde{\phi}}{q}$$

$$k^2 = \frac{q^2}{m\epsilon_0}\int_{-\infty}^{\infty}\frac{\partial f_0/\partial v}{v-\omega/k}$$

v_{ph} singularity.

This is dispersion relation.



form of $f_0(v)$ & integrand

devide into two parts

(a) central region, dominated by $\partial f_0/\partial v$

(b) region near $v = v_{ph}$ dominated by singularity.

(a) assuming / remember $v \ll v_{ph}$

The integrand is

$$-\frac{1}{v_{ph}} \int \frac{\partial f_0/\partial v}{1 - v/v_{ph}} \, dv \approx -\frac{1}{v_{ph}} \int \frac{\partial f_0}{\partial v} \left\{ 1 + \frac{v}{v_{ph}} + \left(\frac{v}{v_{ph}}\right)^2 + \left(\frac{v^3}{v_{ph}}\right) \ldots \right\} dv$$

$\Rightarrow$ due to antisymmetry of $\partial f_0/\partial v$ even power are zero.

$$= -\frac{1}{v_{ph}^2} \int v \frac{\partial f_0}{\partial v} \, dv - \frac{1}{v_{ph}^4} \int v^3 \frac{\partial f_0}{\partial t} \, dv$$

① ②

Term ① $\Rightarrow$

$$-\frac{1}{v_{ph}^2}\left(\left[v f_0\right]_{-\infty}^{\infty} - \int f_0\, dv\right) = \frac{n}{v_{ph}^2}$$

Term ②

$$-\frac{1}{v_{ph}^2}\left(\left[v^3 f_0\right]_{-\infty}^{\infty} - 3\int f v^2 dv\right) = \frac{3}{v_{ph}^4}\, n\overline{v}^2$$

combining

$$k^2 = \frac{q^2 n}{m\epsilon_0}\left(\frac{k^2}{\omega^2} + \frac{3k^4}{\omega^4}\overline{v^2}\right) \quad \text{— for (a)}$$

now $\frac{q^2 n}{m\epsilon_0} = \omega_p^2$

$$\therefore \omega^2 = \omega_p^2 + 3\frac{\omega_p^2}{\omega^2} k^2 \overline{v}^2$$

$\omega \simeq \omega_p$

$$\omega^2 = \omega_p^2 + 3k^2\overline{v}^2 \Rightarrow v \sqrt{\frac{k_B T_e}{m_e}}$$

Bohm Gross ~~d~~dispersion Relation.

$$\omega = \omega_{pe}^2 + \frac{3k_B T_e}{m_e} k^2$$

Singularity

We are now concerned only with arbitrary narrow range of velocity near $v - \omega/k = 0$

we regard $\partial f_0/\partial v$ as constant over this narrow range + move outside integral.

$v - \omega/k = u$ we get

$$k^2 = \frac{q^2}{m\epsilon_0}\left(\frac{\partial f_0}{\partial v}\right)_{\omega/k} \int_{-\infty}^{\infty} \frac{du}{u}$$

$$\int_{-a}^{\infty} \frac{du}{u} = \ln(a) - \ln(-a)$$

$$= \ln(a) - \ln\left(a\, exp(i\pi + i2\pi n)\right)$$

$$= \ln(a) - \ln(a) - (i\pi + 2\pi n)$$

$$= -i\pi \qquad n = 0.$$

$$k^2 = -\frac{iq^2\pi}{m\epsilon_0}\left(\frac{\partial f}{\partial v}\right)_{\omega/k}$$

Both contributions, we have

$$k^2 = k^2\left(\frac{\omega_p}{\omega}\right)^2\left(1 + 3\frac{k^2}{\omega^2}\bar{v}^2\right) - i\pi\frac{\omega_p^2}{n}\left(\frac{\partial f_0}{\partial v}\right)_{\omega/k}.$$

imaginary part describes the damping

neglecting $\overline{v^2}$

$$k^2 = k\left(\frac{\omega_p}{\omega}\right)^2 - i\pi \frac{\omega_p^2}{n}\left[\frac{\partial f_0}{\partial v}\right]_{\omega/k}$$

or

$$\omega = \omega_p\left[1 - \frac{i\pi\omega_p^2}{2k^2 n}\left(\frac{\partial f}{\partial v}\right)\right]$$

using Binomial expansion.

~~et~~ $\exp(-i\omega t)$ becoms

$$\exp(-i\omega_p t)\,\exp\left(\frac{\pi\omega_p^2}{2k^2 n}\frac{\partial f_0}{\partial t}\,t\right)$$

negative at $v = \omega/k$.

This describes damping even in absence of collision or dissipitive ~~fo~~ mechanism.