Two fluid eq^n

$$Mn\left[\frac{\partial \vec{u}_i}{\partial t} + (\vec{u}_i \cdot \nabla)\vec{u}_i\right] = en(\bar{E} + \bar{u}_i \times \bar{B}) - \nabla p_i + \vec{P}_{ie}$$

$$mn\left[\frac{\partial u_e}{\partial t} + (\bar{u}_e \cdot \bar{\nabla})\bar{u}_e\right] = -en(\bar{E} + \bar{u}_e \times \bar{B}) - \nabla p_e + \bar{P}_{ei}$$

We obtain two powerful relation by taking linear combination

First, by adding two eq^n,

$$n\left(\frac{\partial}{\partial t}(M\vec{u}_i + m\bar{u}_e) + M(u_i \cdot \nabla)u_i + m(\bar{u}_e \cdot \bar{\nabla})u_e\right)$$

$$= en(\bar{u}_i - \bar{u}_e) \times \bar{B} - \nabla(p_i + P_e)$$

We note $P_{ei} = -P_{ie}$

$$\rho\left(\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\bar{v}\right) = \vec{J} \times \bar{B} - \nabla p + \vec{F}$$

for $m \ll M$, here $\vec{F}$ is any external generalized force.

Momentum conservation or force balance for neutral plasma.

Second we multiply ion eqn by m & electron eqn by M & Jubstract .

$$Mmn\left(\frac{\partial}{\partial t}(u_i - u_e) + (\bar{u}_i \cdot \bar{\nabla})u_i - (\bar{u}_e \cdot \nabla)\bar{u}_e\right)$$

$$= en\,(M+m)\,\vec{E} + en(m\vec{u}_i + M\bar{u}_e)\times\bar{B}$$

$$- m\,\bar{\nabla}p_i + M\nabla p_e - (M+m)\,P_{ei}$$

neglect convective terms on the left hand side which is a common case → restrict ourself to small velocities. therefor u_i & u_i quadratic better is if we take relative magnitude.

again with $\bar{J}$, $m \ll M$

$$\frac{Mmn}{e}\frac{\partial}{\partial t}\left(\frac{\bar{J}}{n}\right) = e\rho\vec{E} - Mne\eta\vec{J} + M\bar{\nabla}p_e$$

$$+ en\,(m\bar{u}_i + M\bar{u}_e)\times\bar{B}$$

The last term can be simpified

$$m\bar{u}_i + M\bar{u}_e = M\bar{u}_i + mu_e - (M-m)\,\bar{u}_i - \bar{u}_e$$

$$\simeq \frac{\rho}{n}\vec{v} - \frac{M}{ne}\vec{J}$$

dividing by $e\rho$ & rearranging

$$\frac{m}{ne^2}\frac{\partial \bar{J}}{\partial t} = \bar{E} + (\bar{U} + \bar{B}) - \frac{1}{ne}(\bar{J} \times \bar{B}) + \frac{1}{ne}\nabla p_e - \eta \vec{J}$$

This is generalized Ohm's law with each term emf (Volt/m) left hand side descri electron inertia & only matters for high frequency phenomena.

In case of uniform collisionless plasma without magnetic field

$$\frac{m}{ne^2}\frac{\partial \bar{J}}{\partial t} = \bar{E} \qquad \bar{J} = ne\bar{v}$$

we get newton's law

$$m\frac{\partial \bar{u}}{\partial t} = e\bar{E}$$

$\bar{J} \times \bar{B} \Rightarrow$ Hall effect

$\nabla p_e \Rightarrow$ electron pressure gradien that drives the currents.

The MHD eq^n

$$\frac{\partial \rho}{\partial t} + \bar{\nabla}\cdot(\rho \bar{v}) = 0$$

$$\rho \frac{d\bar{v}}{dt} = (\bar{J} \times \bar{B}) - \nabla p$$

$$\frac{d}{dt}\left(P/\rho^{\gamma}\right) = 0$$

$$\bar{E} + (\bar{u} \times \bar{B}) = \eta \vec{J}$$

resistivity.

& Maxwell's eq^n

$$(\nabla \times \bar{E}) = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \vec{J}$$

Ideal MHD = no resistivity

$$(E + \bar{u} \times \bar{B}) = 0$$

Momentum eq^n

$$\rho \frac{\partial \bar{u}}{\partial t} = \bar{J} \times \bar{B} - \nabla p$$

in steady state

$$\frac{\partial \bar{u}}{\partial t} = 0$$

$$\therefore \quad \underbrace{\bar{J} \times \bar{B}}_{\text{Lorentz force}} = \nabla p$$

force due to plasma pressure gradient

$$\nabla \times \bar{B} = \mu_0 \bar{J}$$

$$\nabla p = \frac{1}{\mu_0} (\nabla \times \bar{B}) \times \bar{B} = \frac{1}{\mu_0} \left[(\bar{B} \cdot \bar{\nabla}) \bar{B} - \frac{1}{2} \bar{\nabla} B^2 \right.$$

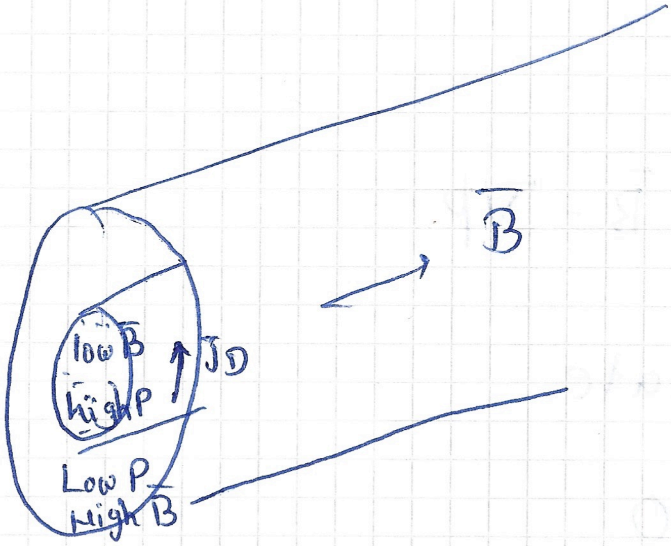
$$\nabla \left(p + \frac{B^2}{2\mu_0} \right) = \frac{1}{\mu_0} (\bar{B} \cdot \nabla) \bar{B}$$

magnetic Plasma

if $\bar{B}$ does not chang dir.^{n} $\therefore \nabla \cdot \bar{B} = 0$

RHS $= 0$

$$p + \frac{B^2}{2\mu_0} = \text{const.}$$



Diagmagnetic field reduces $\overline{B}$

Fusion Science . $\beta = \dfrac{\text{particle pressure}}{\text{magnetic pressure}}$

$$= \frac{P}{\frac{B^2}{2\mu_0}}$$

p = all particle pressures.

$$\beta = \frac{\sum n k_B T}{B^2/2\mu_0}$$

Tokamak $\beta < 1$

magnetic pressure > particle pressure