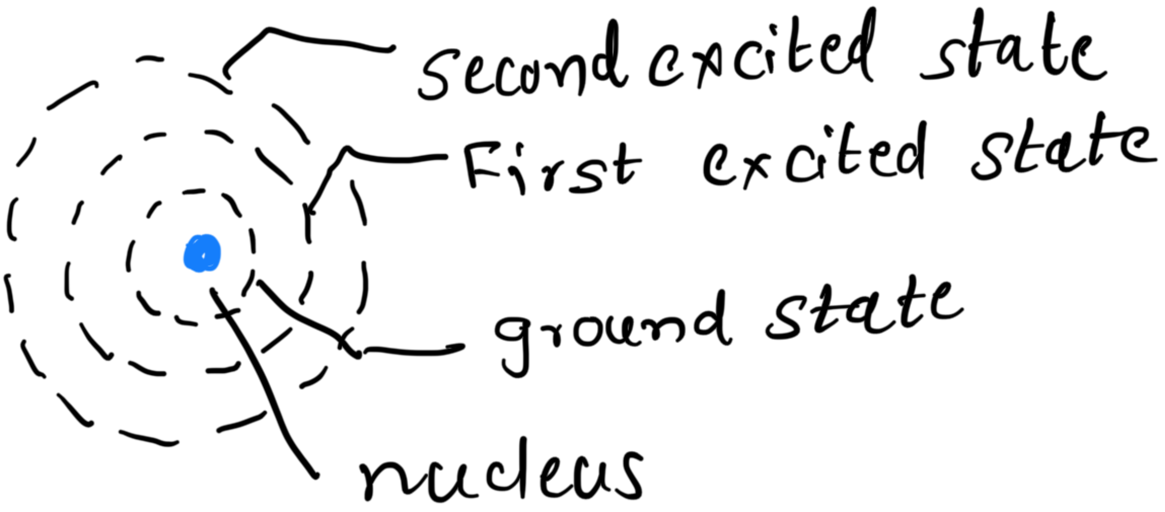
# Quantum Mechanics & Quantum Computing

## Qubits & Uncertainty principle




Energy of electron in an atom is quantized & certain energy levels.


ground state $\leftrightarrow$ 0
excited state $\leftrightarrow$ 1

electron is partly in ground & partly in excited state

Ground with $P = 1/3$
excited state $P = 2/3$

Superposition of state with complex amplitude:

$\alpha\,|0\rangle$ ground

$\beta\,|1\rangle$ excited

example:

Example: $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$|\alpha|^2 + |\beta|^2 = \underline{1}$ state must be normalized

amplitude is allowed to be complex

$$\left(\frac{1}{2} + \frac{1}{2}i\right)|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$\alpha = a + bi \qquad |\alpha| = \sqrt{a^2+b^2}$

$|\alpha|^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} \qquad |\beta|^2 = \frac{1}{2}$

if we "look" at it

measurement:

ground state: $|0\rangle \quad P = |\alpha|^2$

$|1\rangle \quad P = |\beta|^2$

measurement disturbs the state & electron is then either state. Thus we need state to be normalized so that

$|\alpha|^2 + |\beta|^2 = \underline{1}$

Thus two complex number gives superposition $\Rightarrow$ large amount of information or infinite number of bits infinite ...

Geometric interpretation:

state $\underline{\alpha|0\rangle + \beta|1\rangle}$ Ket Notation Dirac

$\alpha, \beta \in \mathbb{C}$

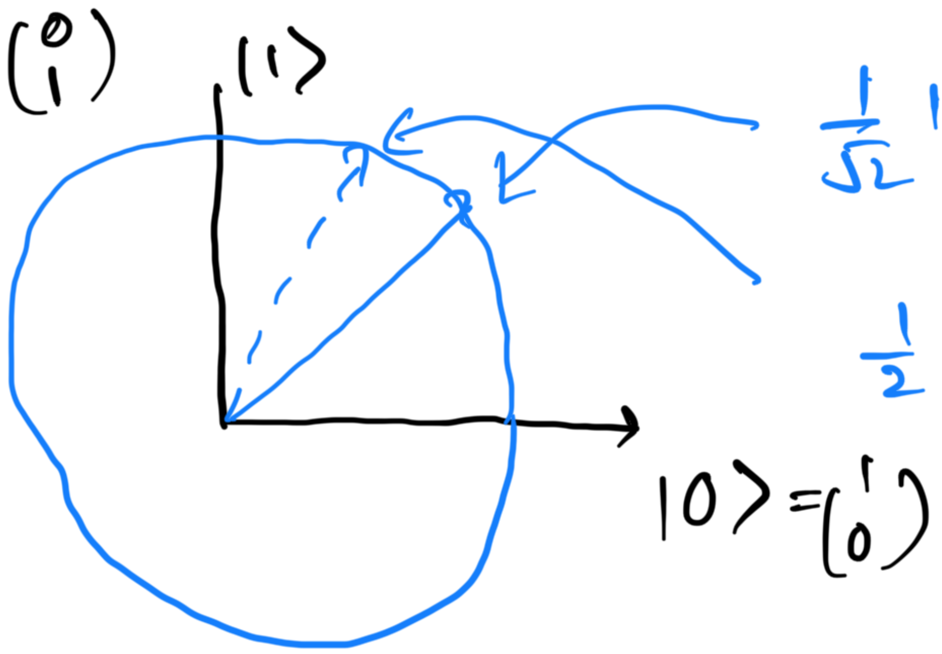
$$|\alpha|^2 + |\beta|^2 = 1$$

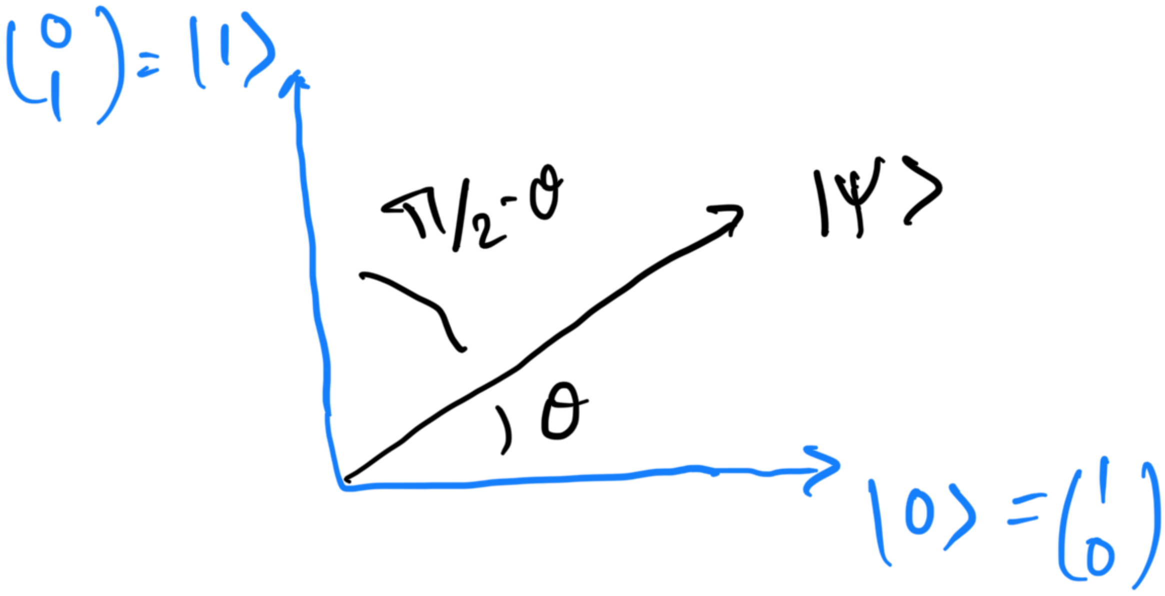
we can also write

$\begin{pmatrix}\alpha \\ \beta\end{pmatrix} \longleftrightarrow$ vector space
2D complex
Unit vector

$|0\rangle = \begin{pmatrix}1 \\ 0\end{pmatrix}$ $\quad$ $|1\rangle = \begin{pmatrix}0 \\ 1\end{pmatrix}$

$\alpha = 1 \;\; \beta = 0$ $\qquad$ $\alpha = 0 \;\; \beta = 1$




$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle = \begin{pmatrix}\cos\theta \\ \ldots\end{pmatrix}$

$\begin{pmatrix} \\ \sin\theta \end{pmatrix}$

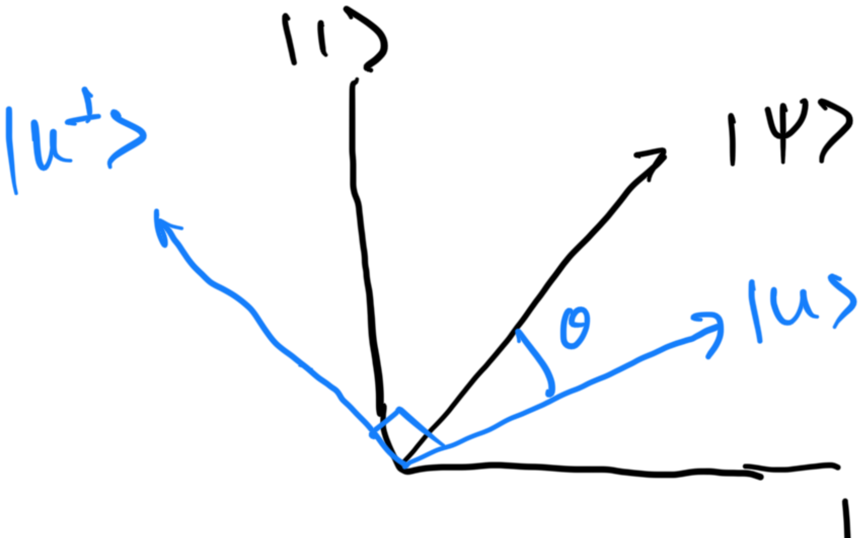
Measure:

$|0\rangle$ with $p = \cos^2\theta$

$|1\rangle$ with $p = \sin^2\theta$

$\Downarrow$

$\cos^2\left(\frac{\pi}{2} - \theta\right)$




measure $|\psi\rangle$ in $u, u^\perp$ basis

$|u\rangle \; : \; p = \cos^2\theta$

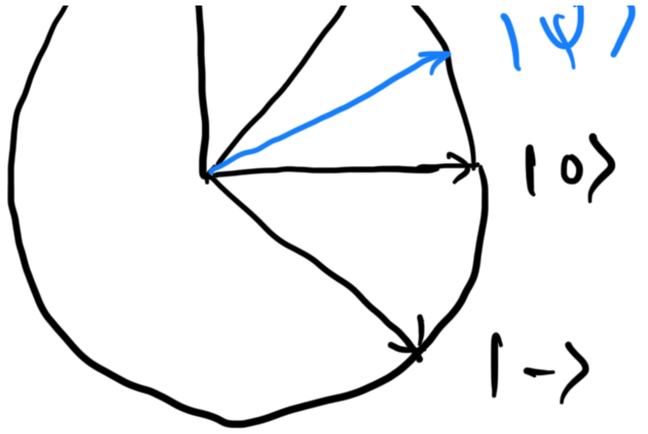
$|u^\perp\rangle \; : \; p = \sin^2\theta$

$$|u\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|u^\perp\rangle = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

electron is in excited or ground state but rather trying to find $|u\rangle$ or $|u^\perp\rangle$ state

$|0\rangle, |1\rangle$




$$\left.\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ |-\rangle &= \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{aligned}\right\} \text{sign}$$

bit $\leftrightarrow$ position

sign $\leftrightarrow$ momentum

Can we know both bit & sign of qubit simultaneouly

Bit value known perfectly:

$|0\rangle$ or $|1\rangle$

Sign value

$|+\rangle$ or $|-\rangle$

if $|\psi\rangle$ tries to be close to either $|0\rangle$ or $|1\rangle$ farther it goes from $|+\rangle$ & $|-\rangle$

simpler words for uncertainty principle

Spread of a quantum state

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \beta_0|+\rangle + \beta_1|-\rangle$$

Spread in standard basis

$$S(|\psi\rangle) = |\alpha_0| + |\alpha_1$$

Spread in sign basis

$$\hat{S}(|\psi\rangle) = |\beta_0| + |\beta_1|$$

$$S(|0\rangle) = 1+1 = 1 \qquad \hat{S}(|0\rangle) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$S(|+\rangle) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \qquad \hat{S}(|+\rangle) = 1+0 = 1$$

Uncertainty principle

$$S(|\psi\rangle)\,\hat{S}(|\psi\rangle) \geq \sqrt{2}$$

for any $|\psi\rangle$.