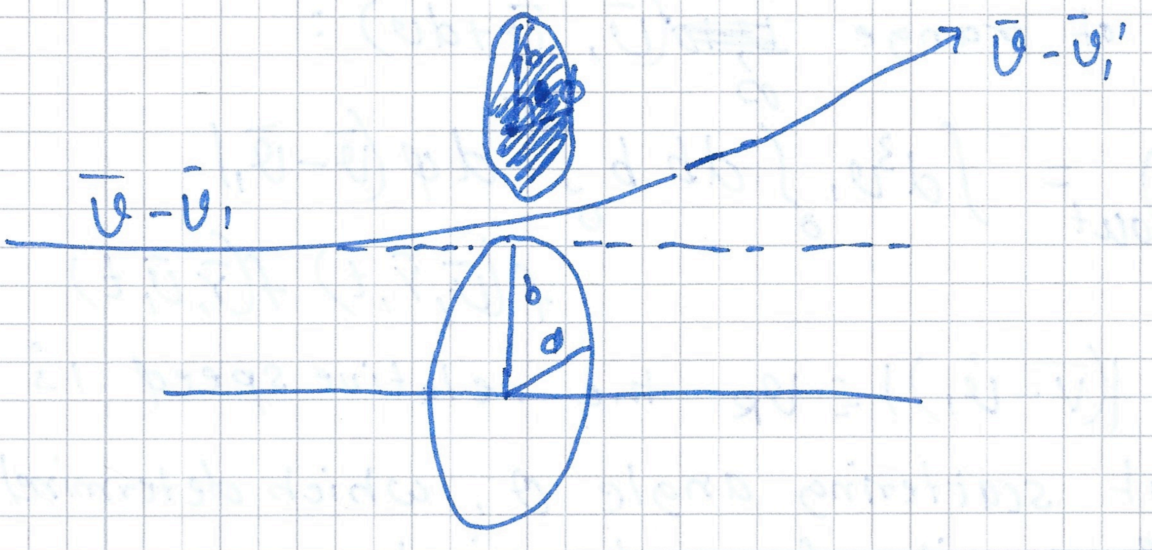
$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial r_i} + a_i \frac{\partial f}{\partial v_i} = \left(\frac{\delta f}{\delta t}\right)_c$$

Collision term $(\delta f / \delta t)_c$

Two body collision : before & after collision the particle velocities are $(\bar{v}, \bar{v}_1)$ & $(\bar{v}', \bar{v}_1')$ where primed are after collision. Convienent to describe in center-of-mass frame, collisin is equivalent to dettection of fictitous particle with reduced mass (moving with relative velocity) from scattering center.



Incident particle approaches the scattering center with impact parameter b & deflected by scattering angle θ. Orbit of particle involves two angles (ϕ, θ) where ϕ is azimuthal & θ is polar angle corresponding to scattering angle flux of particles passing through an elemental area $b\,db\,d\phi$ is then

$$b\,db\,d\phi\,(\bar{v}-\bar{v}_1)\,f(\bar{r},\bar{v}_1,t)\,d^3v_1 \quad (sec^{-1})$$

Then number of collisons (per unit time) of particle in above flux with particles of velocity $\bar{v}$ is times $f(\bar{v},\bar{r},t)\,d^3\bar{v}$

$$b\,db\,d\phi\,|\bar{v}-\bar{v}_1|\,f(\bar{r},\bar{v}_1,t)\,d^3v_1\,f(\bar{v},\bar{r},t)\,d^3v$$

Integrating this over all $\bar{v}_1$ & deviding by $d^3\bar{v}$ gives total rate of change of $f(\bar{v},\bar{r},t)$ due to collisions which scatter $\bar{v}$ - particles out of range ~~$\bar{v}$~~ $(\bar{v},\bar{v}+dv)$:

$$\left(\frac{\delta f}{\delta t}\right)_{out} = \int d^3v_1 \int_0^\infty db\, b \int_0^{2\pi} d\phi\,|\bar{v}-\bar{v}_1|\, f(\bar{v},\bar{r},t)\,f(\bar{r},\bar{v}_1,t)$$

Note $|(\bar{v}-\bar{v}_1)| = v_R$ the relative speed is fu^n of scattering angle θ, which determinel by b & φ if force law is known.

it is customary to introduce differential scattering cross section

$$d\sigma(v_R,\theta) = b\,db\,d\phi \qquad \theta = \theta(v_R, b)$$

In to the center of of mass frame θ is Fu^n of b & initial relative velocities

$$\left(\frac{\delta f}{\delta t}\right)_{out} = \int d^3 v_1 \int d\Omega \frac{d\sigma}{d\Omega} |\bar{v} - \bar{v}_1|$$

$$f(\bar{v}, \bar{r}, t) f(\bar{r}, \bar{v}_1, t)$$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

rate of scattering into range $(\bar{v}, v + d\bar{v})$ is then

$$\left(\frac{\delta f}{\delta t}\right)_{in} = \int d^3 v_1' \int \left(d\Omega \frac{d\sigma}{d\Omega}\right)' |\bar{v}' - \bar{v}_1'|$$

$$f(\bar{r}, \bar{v}', t) f(r, \bar{v}_1', t)$$

for elastic collisions

$$m\bar{v}' + m\bar{v}_1' = m\bar{v} + m\bar{v}_1$$

$$\frac{m}{2}\bar{v}'^2 + \frac{m}{2}\bar{v}_1'^2 = \frac{m}{2}\bar{v}^2 + \frac{m}{2}\bar{v}_1^2$$

collision can be thought of linear orthogonal transformation from $(\bar{v}, \bar{v}_1)$ to (v', v_1')

$$|\bar{v} - \bar{v}_1| = |v' - v_1'|$$

symmetry of interaction

$$\left(\frac{d\sigma}{d\Omega} d\Omega\right)' = \frac{d\sigma}{d\Omega} d\Omega$$

Therefore

$$\left(\frac{\partial f}{\partial t}\right) + v_i \frac{\partial f}{\partial x_i} + a_i \frac{\partial f}{\partial v_i} = \left(\frac{\delta f}{\delta t}\right)_c$$

where

$$\left(\frac{\delta f}{\delta t}\right)_c = \int d^3\vartheta_1 \int d\Omega \frac{d\sigma}{d\Omega} |\bar{\vartheta}-\bar{\vartheta}_1|$$

$$\left[f(\bar{r},\bar{\vartheta}_1',t) - f(\bar{r},\bar{\vartheta},t) f(\bar{r},\bar{\vartheta}_1,t) \right.$$
$$\left. f(\bar{r},\bar{\vartheta}',t) \right]$$

Generalized form

$$\frac{\partial f_\alpha}{\partial t} + \vartheta_i \frac{\partial f_\alpha}{\partial r_i} + a_i \frac{\partial f_\alpha}{\partial \vartheta_i} = \sum_\beta C_{\alpha\beta}(f_\alpha f_\beta)$$

α & β refer to species, $C_{\alpha\beta}$ is the collision operator.

Relaxation Model of collision term

In weakly ionized plasma

$$\frac{\delta f}{\delta t} = -\nu \left[f(\bar{r},\bar{\vartheta},t) - f_0(\vartheta) \right]$$

$f_0(\vartheta)$ is suitable Maxwell distribution

& $1/\nu = \tau$ has dimension of time

& called relaxation time

This is known as Bhatnagar, Gross & Krook model.

$f_0(v)$ makes collision integral vanish

* negative sign tells collison reduces the deviation of distribution faction from equilibrium Maxwellian.

assuming external force absent, ~~pla~~ uniform plasma

$$\frac{\partial}{\partial t} f(\bar{v}, t) = -\nu \left[f(\bar{r}, \bar{v}, t) - f_0(v) \right]$$

integrating

$$f(\bar{v}, t) - f_0(v) = \left[f(\bar{r}, \bar{v}, t) - f_0(v) \right] e^{-\nu t}$$