Lecture 7: Retinotopic Maps and Hopfield Networks

7.1 Organization in neural systems

Neural circuits often preserve spatial relationships between sensory input and brain representation. In retinotopic organization, nearby points on the retina tend to map to nearby locations in visual processing areas. Development involves molecular guidance, neural activity, competition, and plasticity; no single simple learning rule explains every aspect.

7.2 Hebbian learning and competition

A common conceptual rule is that connections strengthen when presynaptic and postsynaptic activity are correlated:

$$\Delta w_{ij} = \eta x_i x_j. \tag{1}$$

Here η is a learning rate and x_i, x_j denote activity variables. This basic Hebbian rule needs additional normalization, competition, or bounds to prevent unrestricted growth and to generate useful maps.

A computational analogy is the self-organizing map (SOM), which updates nearby prototype vectors together:

$$\Delta \mathbf{w}_i = \eta h_{ic}(\mathbf{x} - \mathbf{w}_i), \tag{2}$$

where c is the best-matching unit and h_{ic} is a neighborhood function. A SOM is an educational model of topographic organization, not a complete biological theory of retinotopic development.

7.3 Hopfield networks

A Hopfield network consists of recurrently connected binary neurons. In a common convention, $s_i \in \{-1, +1\}$ and the neuron updates according to the sign of its net input:

$$s_i \leftarrow \operatorname{sign}\left(\sum_j w_{ij} s_j - \theta_i\right). \tag{3}$$

With symmetric weights $w_{ij} = w_{ji}$ and no self-couplings, an energy function can be defined:

$$E = -\frac{1}{2}\sum_{i,j} w_{ij} s_i s_j + \sum_i \theta_i s_i. \tag{4}$$

Under asynchronous deterministic updates, changing a single neuron according to its local field cannot increase this energy. The network can therefore relax toward a stable configuration (an attractor).

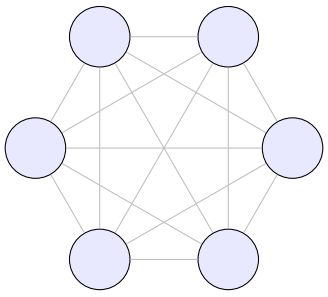


Figure 7. Recurrent connections in a small Hopfield-type network.

7.4 Associative memory

A stored pattern can act as an attractor: if the network begins in a sufficiently similar, noisy state, its dynamics may recover the stored pattern. This is called *content-addressable* or associative memory. Memory performance is limited by interference between patterns, spurious attractors, and network size.

One simple learning prescription is

$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^{P} \xi_i^\mu \xi_j^\mu, \qquad w_{ii} = 0, \tag{5}$$

where ξ^μ are the patterns to be stored. This formula connects Hebbian learning to an energy-based recurrent network.

7.5 Connection to the Ising model

Both the Ising model and a Hopfield network involve interacting binary variables and energy functions. Their interpretation differs: Ising spins are typically studied as a thermal equilibrium ensemble, whereas a Hopfield network is designed to evolve toward attractors encoding memories. The shared mathematical structure helps explain why ideas from statistical mechanics are useful in neural computation.

Key idea. Neural organization can emerge through activity-dependent interactions, while recurrent networks can use stable collective states to represent memories.

Further reading

J. J. Hopfield, “Neural networks and physical systems with emergent collective computational abilities” (1982); T. Kohonen, *Self-Organizing Maps*.