Lecture 4: Lasers as Self-Organizing Systems

4.1 From amplification to collective oscillation

A laser combines a gain medium, energy pumping, and optical feedback. Stimulated emission increases the number of photons in an optical mode, while absorption and cavity losses reduce it. Above threshold, amplification compensates loss and a macroscopic coherent field can develop.

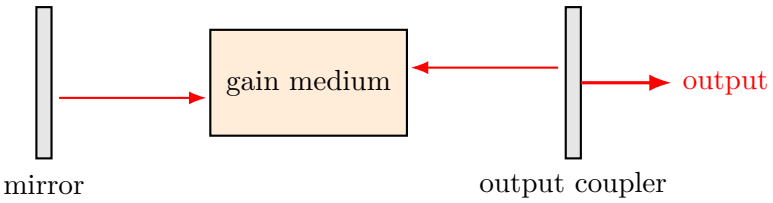


Figure 4. Minimal optical resonator with gain and output coupling.

The term *self-organization* is useful because many emitters contribute to a collective field whose amplitude is determined by feedback and saturation. Nevertheless, the optical cavity also selects resonant modes, and real lasers can exhibit multimode competition and fluctuations.

4.2 A simple rate-equation model

Let S be the intracavity photon number and N a population inversion variable. A minimal model is

$$\frac{dS}{dt} = (GN - \kappa_c)S + \beta R_{\mathrm{sp}}, \tag{1}$$

$$\frac{dN}{dt} = P - \gamma N - GNS. \tag{2}$$

Here G is the gain coefficient, κ_c the photon loss rate, P the pumping rate, γ the inversion relaxation rate, and βR_{sp} the spontaneous-emission contribution into the modeled mode.

If spontaneous emission into the mode is neglected for a simple threshold argument, small S grows when $GN > \kappa_c$. Thus the ideal threshold inversion is

$$N_{\mathrm{th}} = \frac{\kappa_c}{G}. \tag{3}$$

Below threshold, photons are lost faster than they are stimulated into the mode. Above threshold, the photon population rises until stimulated emission depletes the inversion sufficiently to balance the gain.

4.3 Saturation and relaxation

A physical laser does not exhibit unlimited exponential growth: the gain medium has finite stored energy. In steady operation above threshold, the inversion is approximately clamped near its threshold value in the simple model. The photon number then depends on how far pumping exceeds the threshold.

The coupled equations may also support damped relaxation oscillations. In more complex configurations, additional degrees of freedom can produce mode competition, spatial patterns, or irregular temporal dynamics.

4.4 Connection to pattern formation

The laser threshold resembles the onset of convection in one limited sense: a control parameter crosses a critical value and a macroscopic state becomes possible. The underlying physical quantities are different: convection organizes fluid motion, whereas a laser establishes a coherent optical field.

Key idea. Laser oscillation emerges through positive optical feedback, a gain–loss threshold, and nonlinear saturation.

Further reading

H. Haken, *Synergetics: An Introduction*; A. E. Siegman, *Lasers*.