Lecture 2: Chemical Oscillations and Reaction–Diffusion Patterns

2.1 Why can chemical concentrations oscillate?

In a simple reaction approaching equilibrium, concentrations often relax monotonically. In a driven network with autocatalysis and feedback, intermediate concentrations may instead oscillate. The Belousov–Zhabotinsky (BZ) reaction is a classic example: in suitable conditions, the oxidation state of a catalyst changes periodically, and spatially extended samples can show moving chemical wave fronts.

The full BZ chemistry contains many elementary reactions. For pattern formation it is useful to study simplified *activator–inhibitor* descriptions. An activator promotes its own production, while an inhibitor counteracts it on a different timescale.

2.2 Local kinetics and diffusion

Let $u(\mathbf{r}, t)$ and $v(\mathbf{r}, t)$ represent two effective chemical variables. A general reaction–diffusion model is

$$\frac{\partial u}{\partial t} = f(u, v) + D_u \nabla^2 u, \tag{1}$$

$$\frac{\partial v}{\partial t} = g(u, v) + D_v \nabla^2 v. \tag{2}$$

The reaction terms f and g encode local kinetics; the Laplacians describe transport by diffusion. A homogeneous steady state satisfies $f(u_0, v_0) = g(u_0, v_0) = 0$.

A small concentration disturbance may disappear or grow, depending on the kinetic feedback and diffusion rates. One need not calculate the full dispersion relation to understand the central competition: reactions can locally amplify deviations while diffusion spreads them.

2.3 Traveling waves and spiral patterns

Excitable chemical systems have a resting state that can be triggered by a sufficiently strong perturbation. A reaction front travels through the medium and is followed by a refractory region in which immediate re-excitation is suppressed. Curved and broken fronts can rotate, forming spiral waves.

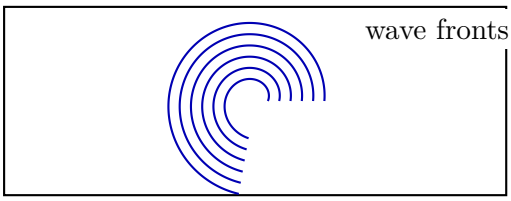


Figure 2. Schematic spiral-like chemical wave fronts; geometry is illustrative.

A wave speed often depends on reaction timescales and diffusion. Dimensional reasoning suggests $c \sim \sqrt{D/\tau}$, where D is a representative diffusivity and τ a reaction time. This is a scaling estimate, not a universal BZ wave-speed formula.

2.4 Turing patterns are a different mechanism

Alan Turing showed that diffusion can destabilize a homogeneous chemical state under suitable kinetic conditions, producing stationary spatial patterns. Such *diffusion-driven instability* often

requires sufficiently different effective diffusion rates. Turing patterns are conceptually distinct from oscillatory BZ waves, although both are studied with reaction–diffusion equations.

A useful classification is:

- **Temporal oscillation:** concentrations change periodically at a fixed position.
- **Traveling wave:** a local disturbance propagates across the medium.
- **Stationary pattern:** a spatially nonuniform concentration profile persists.

2.5 Connection to biology

Reaction–diffusion ideas help model chemical signaling, developmental patterning, and propagating excitation in tissues. A model reproducing a pattern does not by itself prove that the same chemistry operates in a biological organism; the underlying mechanisms must be established experimentally.

Key idea. Local nonlinear chemistry and spatial transport can generate waves, oscillations, and stationary patterns. The outcome depends on feedback, timescales, and diffusion.

Further reading

A. M. Turing, "The Chemical Basis of Morphogenesis" (1952); J. D. Murray, *Mathematical Biology*.