Lecture 1: Introduction and Rayleigh–Bénard Convection

1.1 What is self-organization?

Self-organization is the emergence of large-scale structure from interactions among many components, without a controller specifying the final arrangement. The components may be fluid elements, molecules, oscillators, animals, or neurons. Patterns may be spatial (convection rolls), temporal (chemical oscillations), or collective (synchronized flashing).

An important distinction is between *equilibrium order* (as in a ferromagnet), *driven dissipative structures* (as in a heated fluid), and *forced eigenmodes* (as in a vibrating plate). Similar-looking patterns do not necessarily arise through the same mechanism.

1.2 Feedback, thresholds, and symmetry

A uniform system can be symmetric under translation or rotation. A pattern selects a particular wavelength, position, or orientation and therefore breaks some of that symmetry. Positive feedback amplifies an initial deviation; negative feedback or nonlinear saturation limits it.

A convenient illustration is a scalar amplitude A:

$$\frac{dA}{dt} = \mu A - gA^3, \qquad g > 0. \tag{1}$$

For $\mu < 0$, a small amplitude decays. For $\mu > 0$, the nonzero steady amplitudes $A = \pm\sqrt{\mu/g}$ appear. This is a simple example of a continuous transition; real systems may have additional modes and more complicated transitions. We use this equation as a qualitative model rather than a complete stability theory.

1.3 Rayleigh–Bénard convection

Consider a horizontal liquid layer of thickness d, heated from below and cooled from above. At a sufficiently small temperature difference ΔT, heat is carried mainly by conduction. A warmer fluid parcel is less dense and experiences upward buoyancy; viscosity and thermal diffusion resist its motion. Above a threshold, fluid motion can reinforce the temperature perturbation, producing convection rolls.

The relevant dimensionless control parameter is the Rayleigh number:

$$\boxed{Ra = \frac{g\alpha\Delta T\, d^3}{\nu\kappa}}, \tag{2}$$

where g is gravitational acceleration, α the thermal expansion coefficient, ν the kinematic viscosity, and κ the thermal diffusivity. Large Ra favors buoyancy-driven convection. The Prandtl number $Pr = \nu/\kappa$ compares momentum and thermal diffusion.

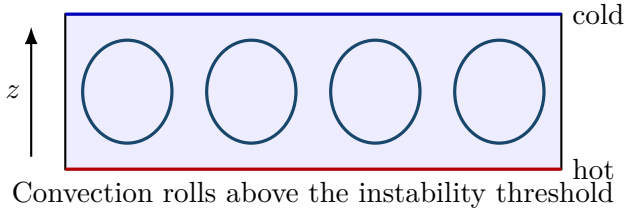


Figure 1. Idealized convection rolls in a layer heated from below.

For a laterally extensive layer with two rigid, isothermal boundaries, the onset occurs at approximately $Ra_c = 1708$. This value is *not universal*: it depends on mechanical and thermal boundary conditions. The initial flow commonly forms rolls; other patterns can emerge depending on geometry, material properties, and nonlinear effects.

1.4 A minimal continuum description

In the Boussinesq approximation, density variations are neglected except where they produce buoyancy. The model combines incompressibility, momentum transport, and heat transport:

$$\nabla \cdot \mathbf{u} = 0, \tag{3}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho_0} + \nu \nabla^2 \mathbf{u} + g\alpha(T - T_0)\hat{\mathbf{z}}, \tag{4}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T. \tag{5}$$

Here $\hat{\mathbf{z}}$ points upward; the buoyancy sign corresponds to this convention. We will not solve these coupled equations in this introductory series. Their role is to show how flow, temperature, and feedback are coupled.

Key idea. A spatial pattern can emerge when a uniform, driven state can no longer suppress small disturbances. The Rayleigh number measures the competition between buoyancy and diffusion.

Further reading

S. H. Strogatz, *Nonlinear Dynamics and Chaos*; M. Cross and H. Greenside, *Pattern Formation and Dynamics in Nonequilibrium Systems.*