Fluid Mechanics

Introduction :

Eq^n of motion for an "ideal fluid"
$\Rightarrow$ Euler in 1755
for "viscous fluid" $\Rightarrow$ Navier (1822)
& stokes (1845)
describe many physical phenomena in nature

Models for atmospheric & ocean flow for weather prediction to applications in chemical engineering, to the development of thrusters

A material exhibits "flow" if shear forces, however small, lead to deformation which is unbounded → definition of a "fluid". A "solid" has fixed shape or atleast limitation on deformation. The main distinguishing feature bet^n two fluids is the notion of compressibility. Gases are usually compressibles, whereas liquids generally incompressible.

Fluids can be subcatagorized "ideal" or "inviscid". The only internal force present is pressure. However fluids can exhibit internal frictional forces involves energy loss - these are "inviscid"

Non-newtonian fluids exhibit reaction to deformation depends on past history eg. paints or temperatur eg. polymers or the size of deformatio eg plastic.

real fluid three natural scales

1. $L_{molecular}$ ⇒ mean free path of collisions
2. L_{fluid} ⇒ medium scale, fluid droplets in pipe, ocean
3 L_{macro} ⇒ scale of fluid geometry scale of container.

$$L_{molecular} \ll L_{fluid} \ll L_{macro}.$$

We assume properties of fluid at L_{fluid} propogate all the way down to molecular level.

This is "continuum assumption".

Basic eq^n based on conservation

principles

1. Conservation of mass
2. Newton's 2^{nd} Law
3. Conservation of energy

These generate

1. Continuity eq^n
2. Navier-Stokes eq^n
3. Equation of state

suppose that fluid is contained within region/domain $D \subseteq R^d$ where $d = 2$ or 3 and $\bar{x} = (x, y, z)^T$ $\in D$ is position or point in D. mainy in Fluid flow field velocity field $\bar{u}(\bar{x}, t) = (u, v, w)^T$.

$$\frac{d}{dt} x(t) = u\big(x(t), y(t), z(t), t\big)$$

$$\vdots \qquad\qquad \vdots$$

The "particle path" or "trajectory" of a fluid is curve traced out by particle.

$$\frac{d}{dt} \bar{x}(t) = \bar{u}\big(\bar{x}(t), t\big)$$

with $x(0) = x_0$ $y(0) = y_0$ $z(0) = z_0$

Suppose for a given fluid flow $\bar{u}(\bar{x}, t)$ we fix time t, then "stream line" is integral curve of $\bar{u}(\bar{x}, t)$ for fixed t ie curve $\bar{x} = \bar{x}(s)$ paramethrized by variable s, that satisfy

$$\frac{d}{ds}\bar{x}(s) = \bar{u}(\bar{x}(s), t)$$

if velocity field $\bar{u}$ is time independent ie $\bar{u} = \bar{u}(\bar{x})$ only $\partial \bar{u}/\partial t = 0$ & flow is called "Stationary"