Radio Frequency Ion Thruster

Principles and modelling

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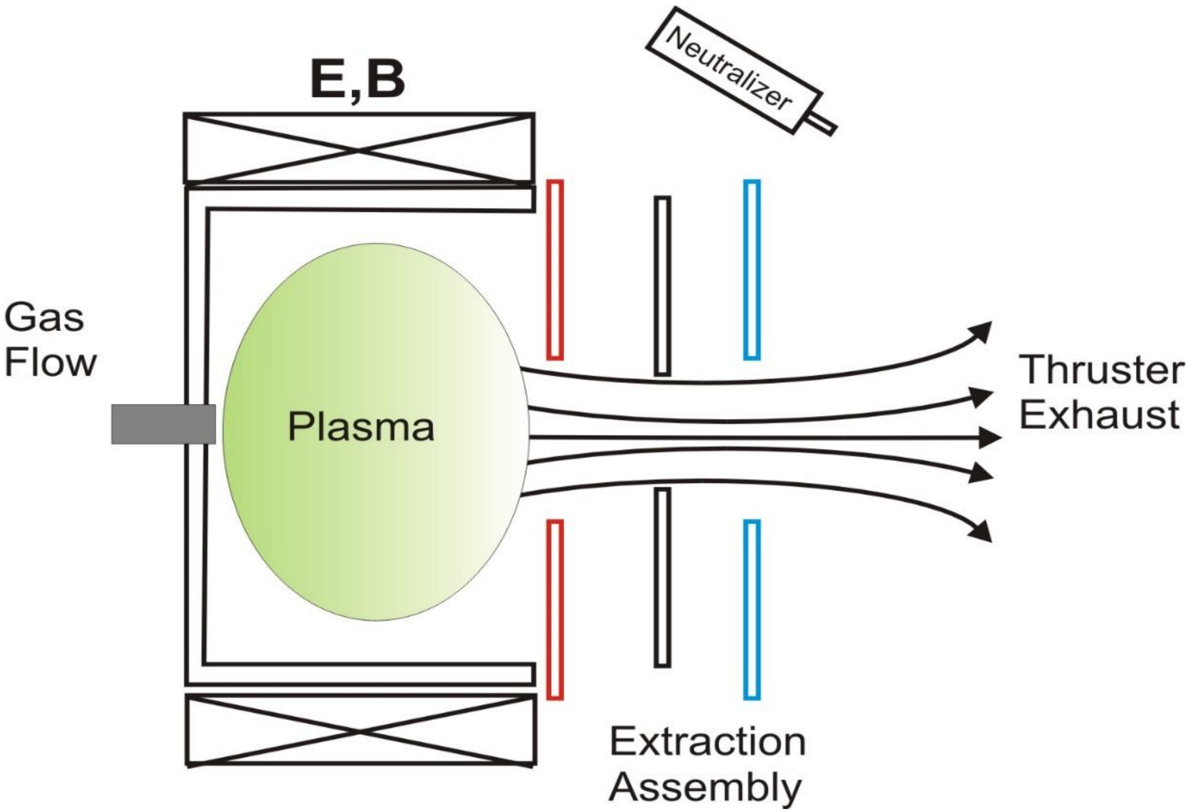
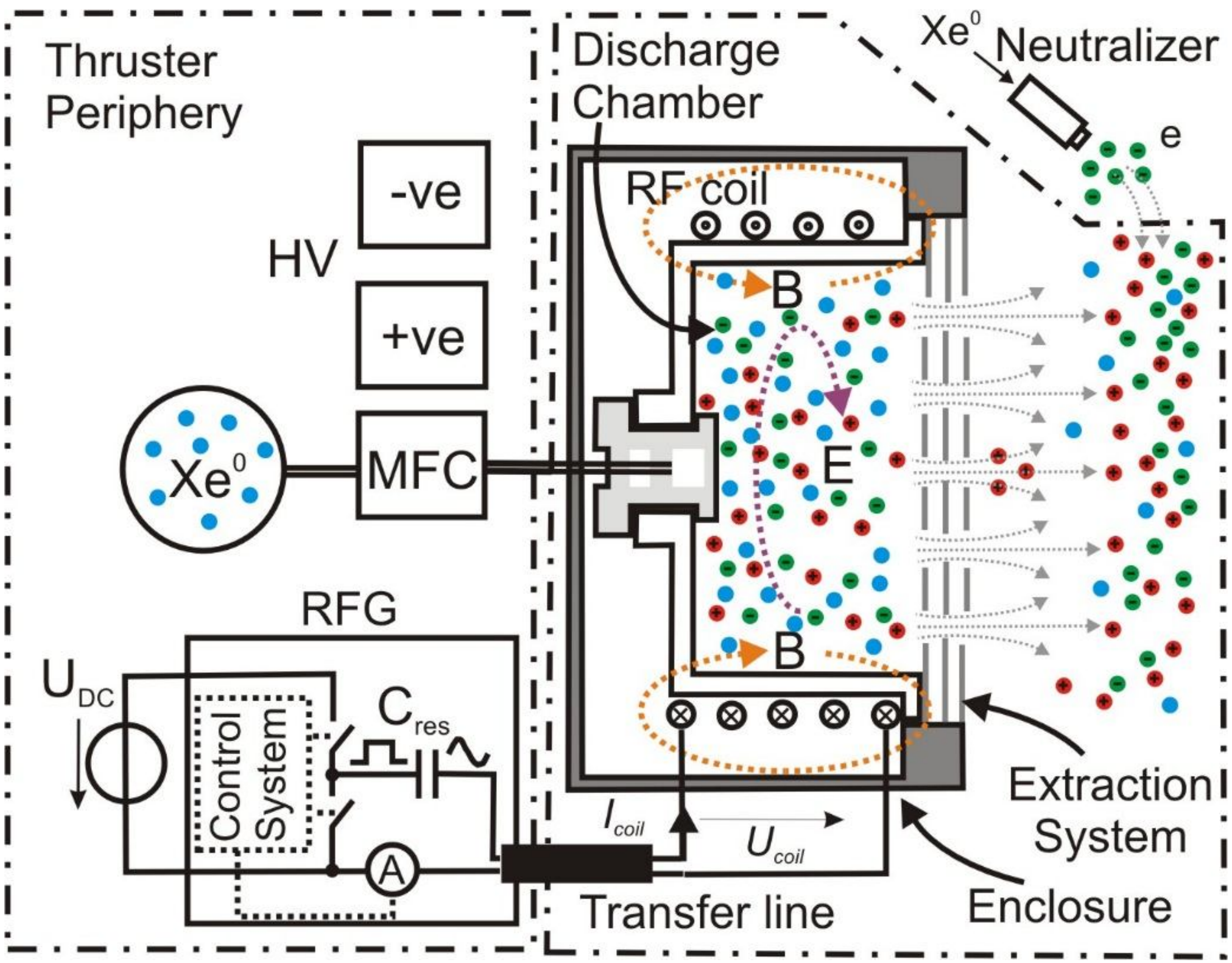
Ion Thruster : Principle

- Working gas → Ignition → Plasma
- Long term confinement can be achieved using external magnetic or electric fields or both
- Using extraction assembly ion are extracted to generate thrust



RF-Ion Thruster

- Based on Inductively coupled plasma source
- It has high I_{sp} ~ 3000-4000 s, scalable beam enery and low divergence angle
- Optimal technology for EOR and GSK



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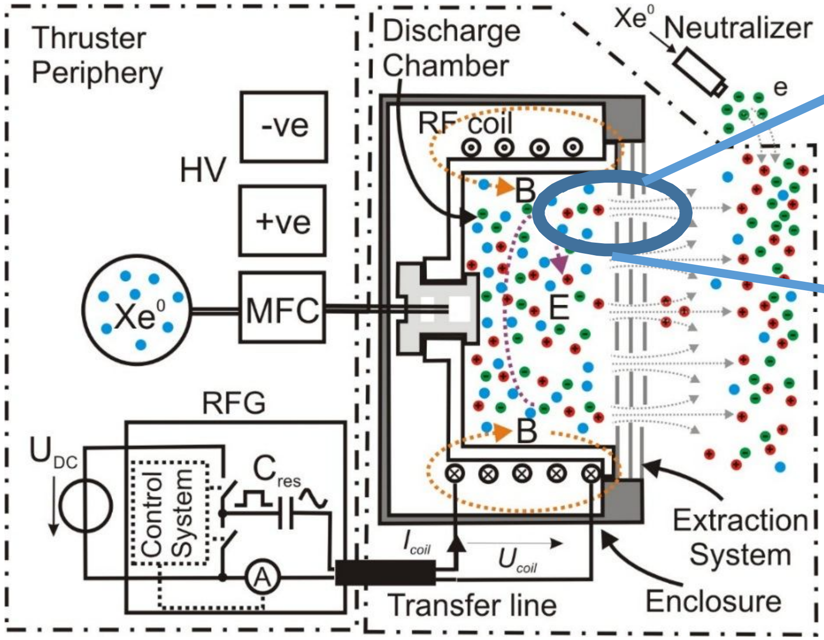
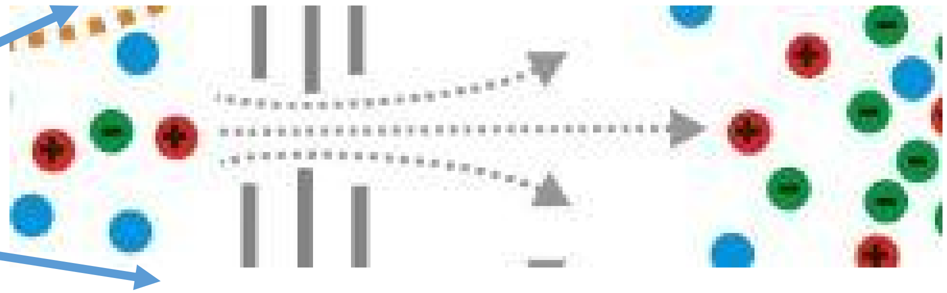
RF-Ion Thruster:





$$I_{sp} = \frac{c_{eff}}{g_0} \qquad I_{sp} = \frac{F}{\dot{m}\, g_0}$$

Specific Impulse I_{sp} relates axial exhaust velocity and mass consumption which overall defines the the figure of merit for the thruster

$$c_{eff} \approx \sqrt{\frac{2 q_i V_{screen}}{m_i}} \cos \eta$$

The grid voltages determine the kinetic energy and hence the exhaust velocity of given ion in terms of q/m ratio; η denotes the divergence angle.

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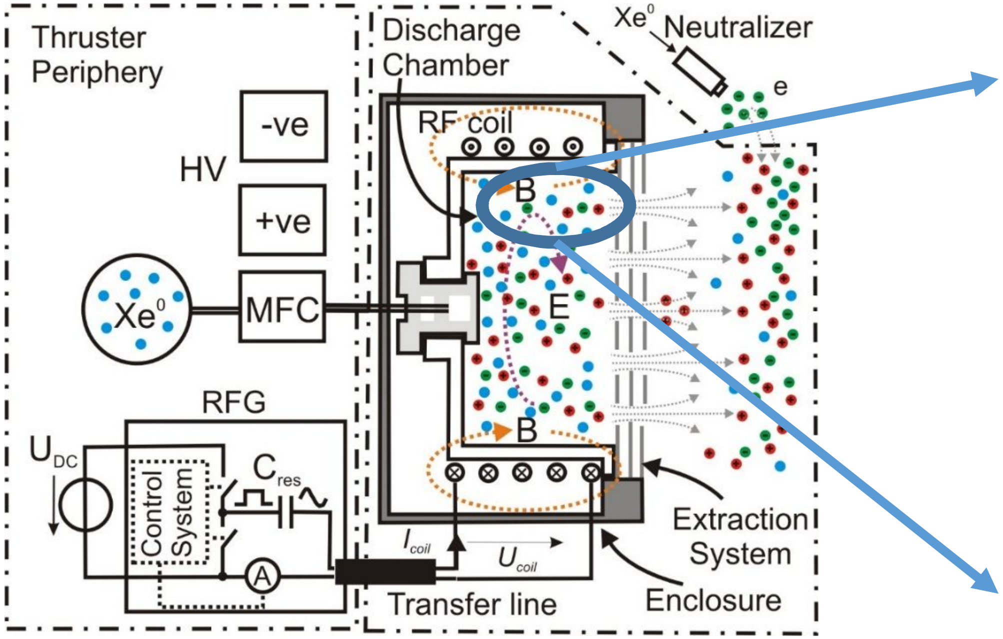
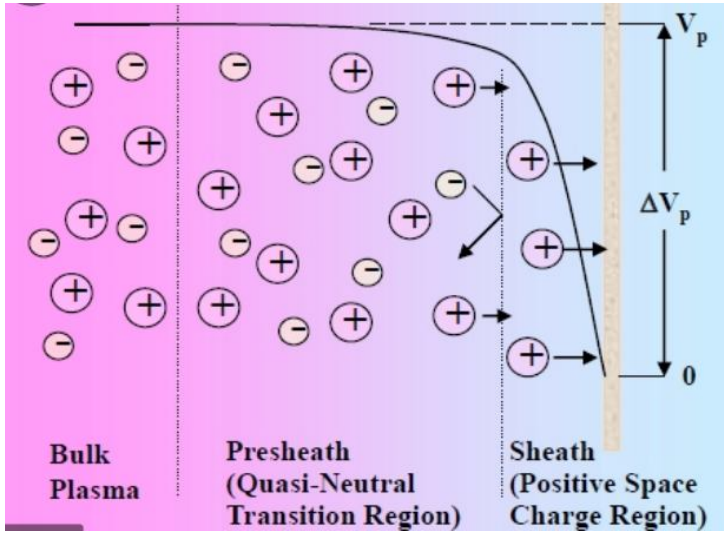
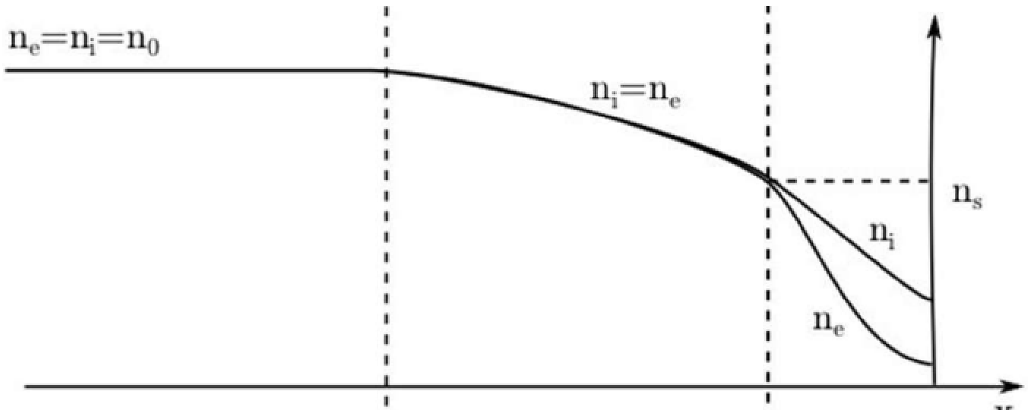
RF-Ion Thruster: Plasma Sheath



Formation of plasma sheath





Extraction of high velocity particles
=> Plasma yield

i.e. Energy and Density at plasma sheath

Thrust is expressed as function of Beam current

$$F = I_{\mathrm{b}} \sqrt{\frac{2 m_{\mathrm{i}} V_{\mathrm{screen}}}{q_{\mathrm{i}}}} \cos \eta.$$

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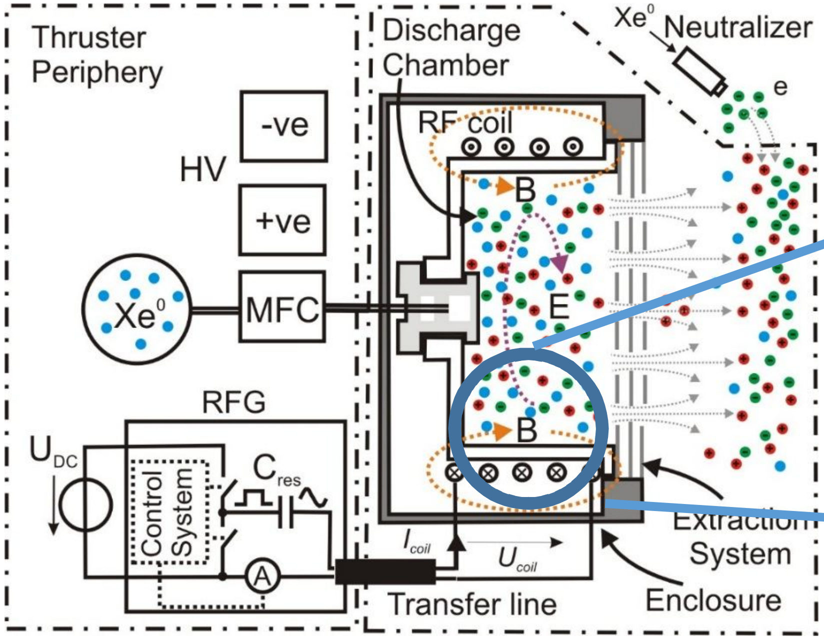
RF-Ion Thruster



Interdependence of plasma parameters, EM fields and input coil current

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \underline{\mathbf{A}}\right) + \mathrm{i}\omega\kappa\underline{\mathbf{A}} + \underline{\mathbf{J}}_{\mathrm{s}} = \mathbf{0}$$

$$\underline{\kappa} \approx \frac{\epsilon_0 \omega_{\mathrm{pe}}^2}{\nu_{\mathrm{eff}}^2 + \omega^2}(\nu_{\mathrm{eff}} - \mathrm{i}\omega)$$

- RF Current
- Frequency
- Mass flow

↔

- Temperature
- Density
- Particle flux

The beam current

$$I_{\mathrm{b}} = e n_{+,\mathrm{s}} u_{\mathrm{B}} A_{\mathrm{g}} \beta_{\mathrm{i}}$$

Bohm velocity

$$u_{\mathrm{B}} = (k_{\mathrm{B}} T_{\mathrm{e}} / m_{\mathrm{i}})^{0.5}$$

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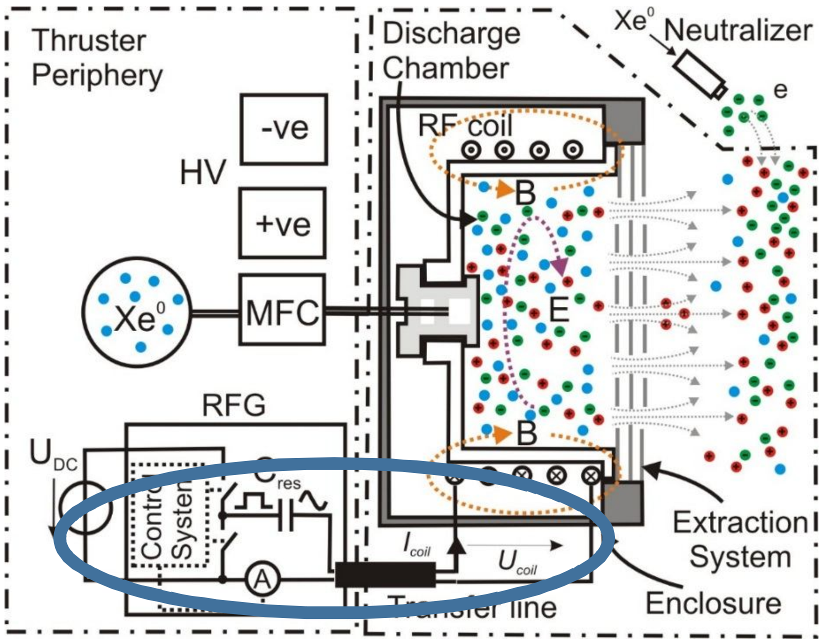
RF-Ion Thruster



Necessary RF Power

$$P_{\mathrm{RF}} = \frac{1}{2}\iiint_V \mathrm{Re}\left\{\underline{\kappa} E^2\right\}\,\mathrm{d}V$$

Efficient energy coupling RF Generator and Thruster

The output frequency is matched with resonant frequency of thruster impedance

Thruster impedance → Propulsion performance ← Input parameters

$$\underline{Z} = R + \mathrm{i}\omega L$$

Poynting theorem

$$\frac{1}{2}\underline{I}^*\underline{V} = \frac{1}{2}\iiint_V \underline{\kappa} E^2\,\mathrm{d}V - \frac{\mathrm{i}\omega}{2\mu_0}\iiint_V B^2\,\mathrm{d}V$$

Ab-initio particle- modelling for Thruster

Levels of Plasma description

Level	Characteristic Parameters
Particle level	$f_{(\vec{r},\vec{v})}$
Fluid level	$\vec{v}_{\mathrm{d}(\vec{r})}, n_{(\vec{r})}$ und $T_{(\vec{r})}$
Global Modelling	n_0 and T_0

Ab-inito particle modelling

- Inductive power coupling for RF-discharge
- Neutral gas distribution calculated through DSMC
- PIC-MCC method include multiple species of ions, electrons and neutrals
- Full 3D description with CAD geometry for asmmetry inclusion
- Calculations are computationally heavy and require HPC

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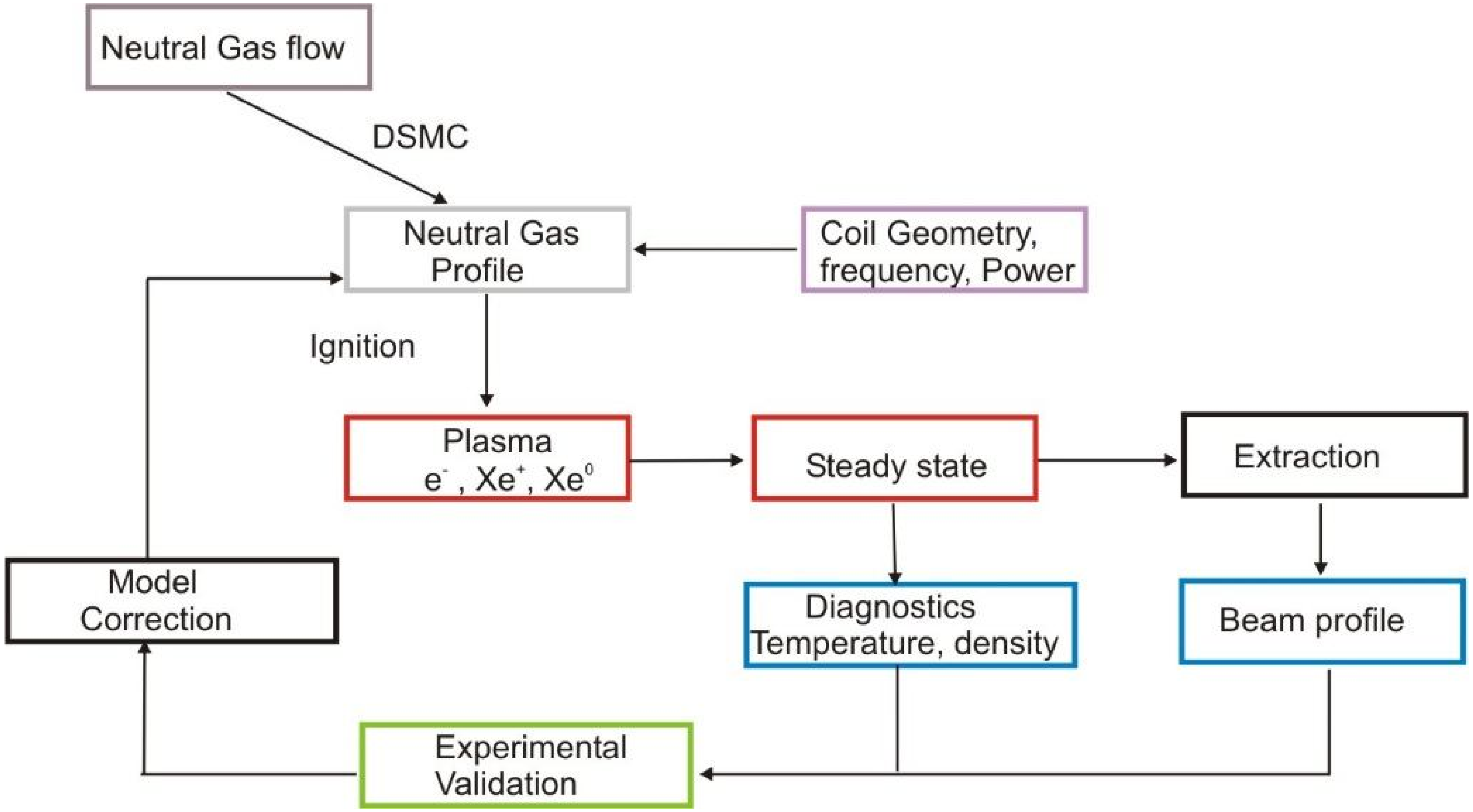
Plasma modelling for Thruster: *ab initio*

- The *ab initio* simulation of plasma inside the thruster can predict the macroscopic quantities electron temperature and ion density profile which can be varified from experiments and can also be used for global modelling
- The extracted ion beam profiles can also experimentally validated



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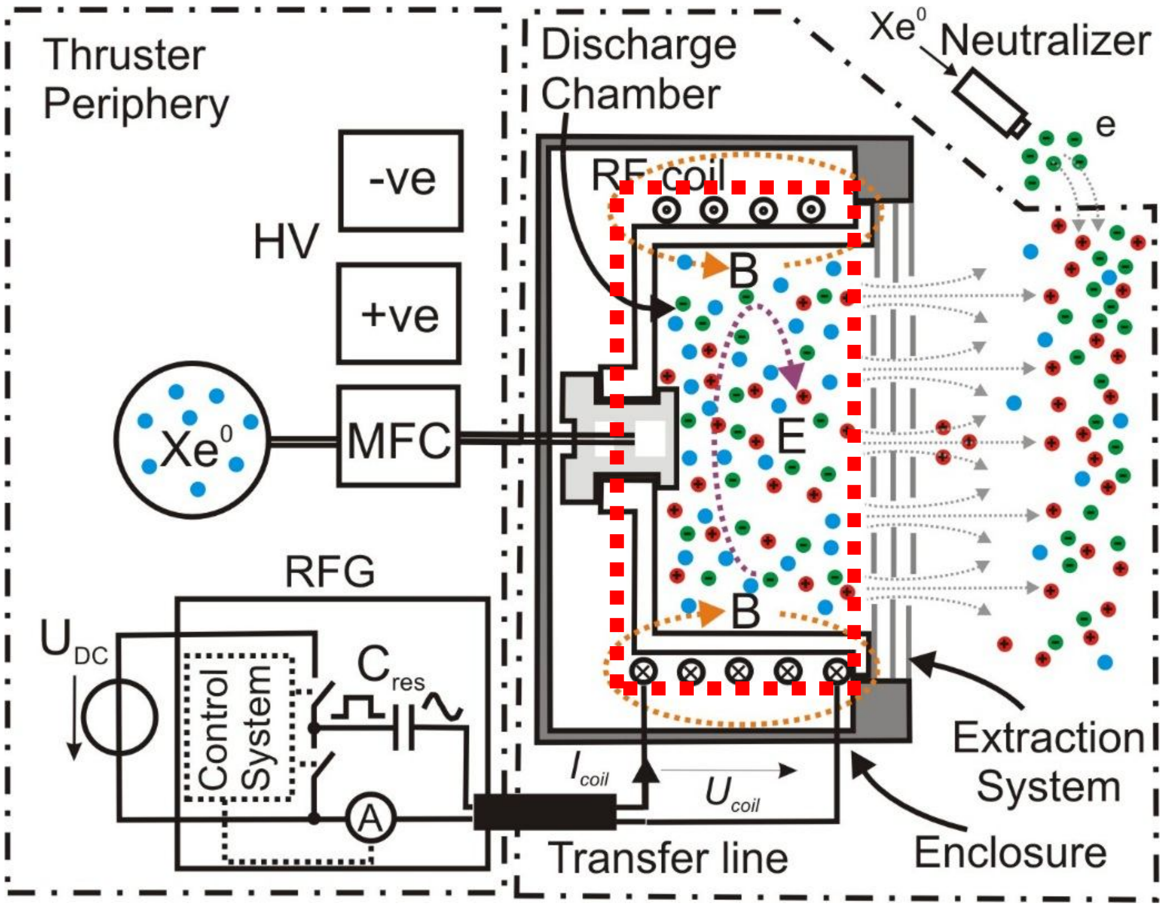
RF Thruster model

- Plasma is generated by inductive heating through coil wound around chamber
- According to Faraday's law varying magnetic field induces electric field,
  $E = -N\ (\Delta\emptyset/\Delta t)$ which drives current density *j* in plasma



- RF frequency, power, gas flow are used as input parameters to calculate ion density and temperature T_e

Plasma Parameters

Plasma Temperature:

From Maxwell-Boltzmann Equation we define temperature of ensemble for particular charged specie

$$T_s = \frac{1}{3k_B} m_s \langle v_s^2 \rangle$$

Plasma Frequency:

Particles are oscillating in the field created by opposite charged particles

$$\omega = \sqrt{\frac{ne^2}{\epsilon_0 m}}$$

Debye Length:

The characteristic length beyond which the potential of charged particle attenuates exponentially

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{ne^2}}$$

Plasma Parameter (Debye Sphere)

Number of particles inside Debye sphere (definition is interchangeable)

$$\Lambda = \frac{4\pi}{3} n \lambda_D^3$$

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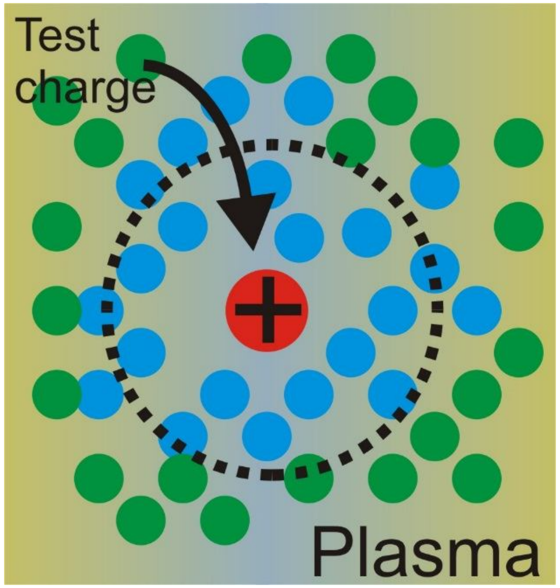
Collective Behaviour: Debye Screening

Consider test charge suspended in plasma creates polarized region



Density distribution must satisfy Poisson‘s equation

$$n(\mathbf{r}) = n_0 \ exp\left(\frac{e\phi(\mathbf{r})}{k_B T}\right)$$

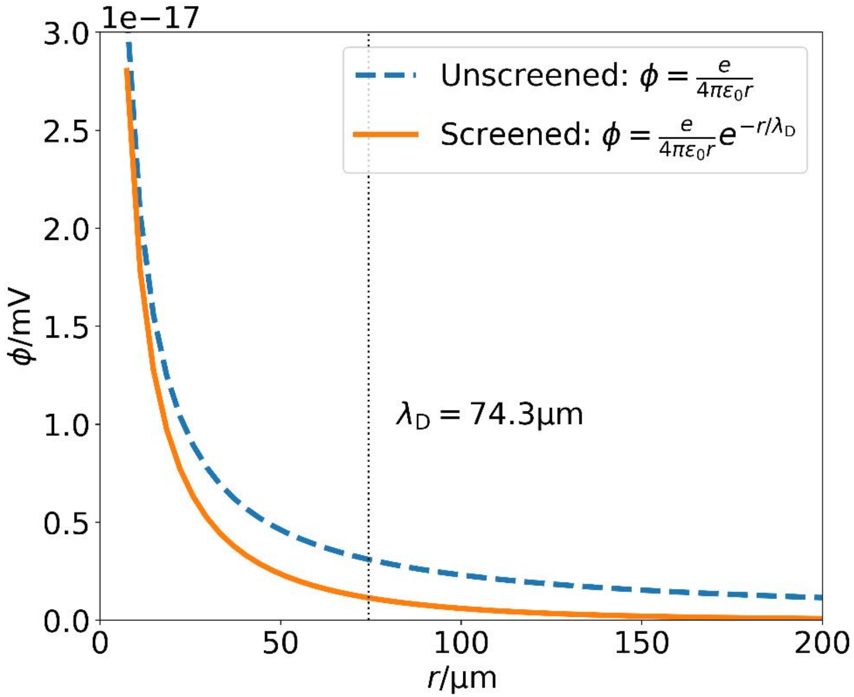
Assuming $q\varphi/k_BT <<1$

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} \simeq \frac{ne^2}{\epsilon_0 k_B T}\phi$$

We get solution in the form

$$\phi(\mathbf{r}) = \frac{Q}{r} \ exp\left(-\frac{r}{\lambda_D}\right)$$

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{ne^2}}$$



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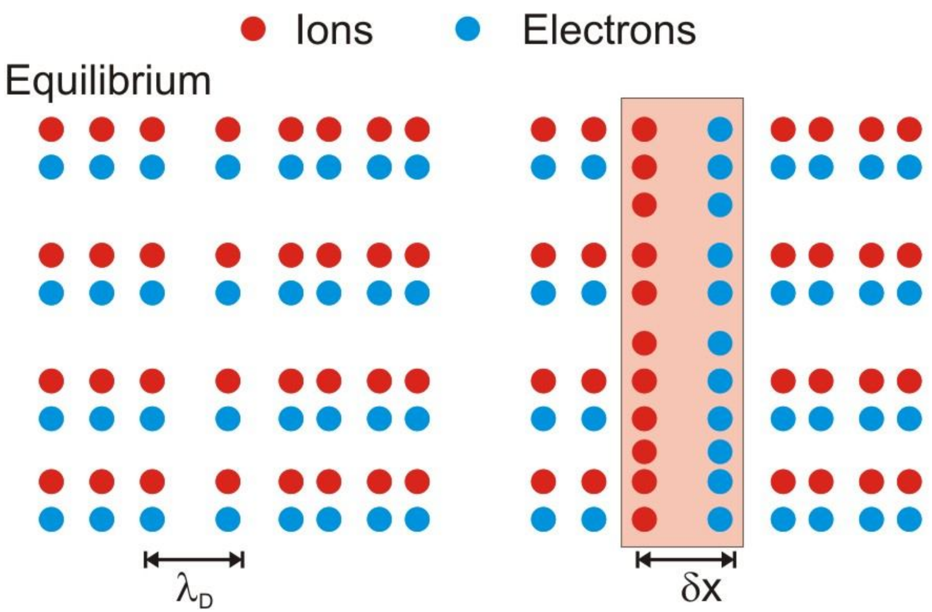
Collective Behaviour: Plasma Oscilations

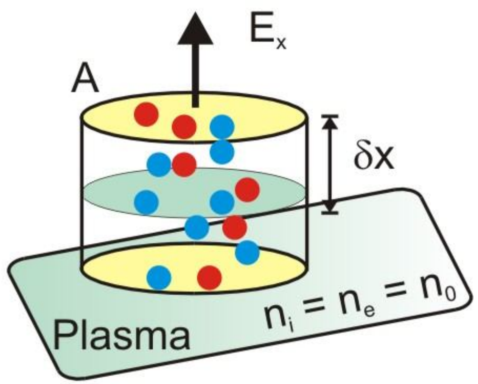
In equilibrium state we have $$n_{0i} \simeq n_{0e} = n_0$$

Electric field due to perturbation $$\int_S \mathbf{E}_1 \cdot d\mathbf{S} = AE_{1x} = \frac{Q}{\epsilon_0} = -\frac{e}{\epsilon_0} n_0 (A\delta x)$$

Equation of motion

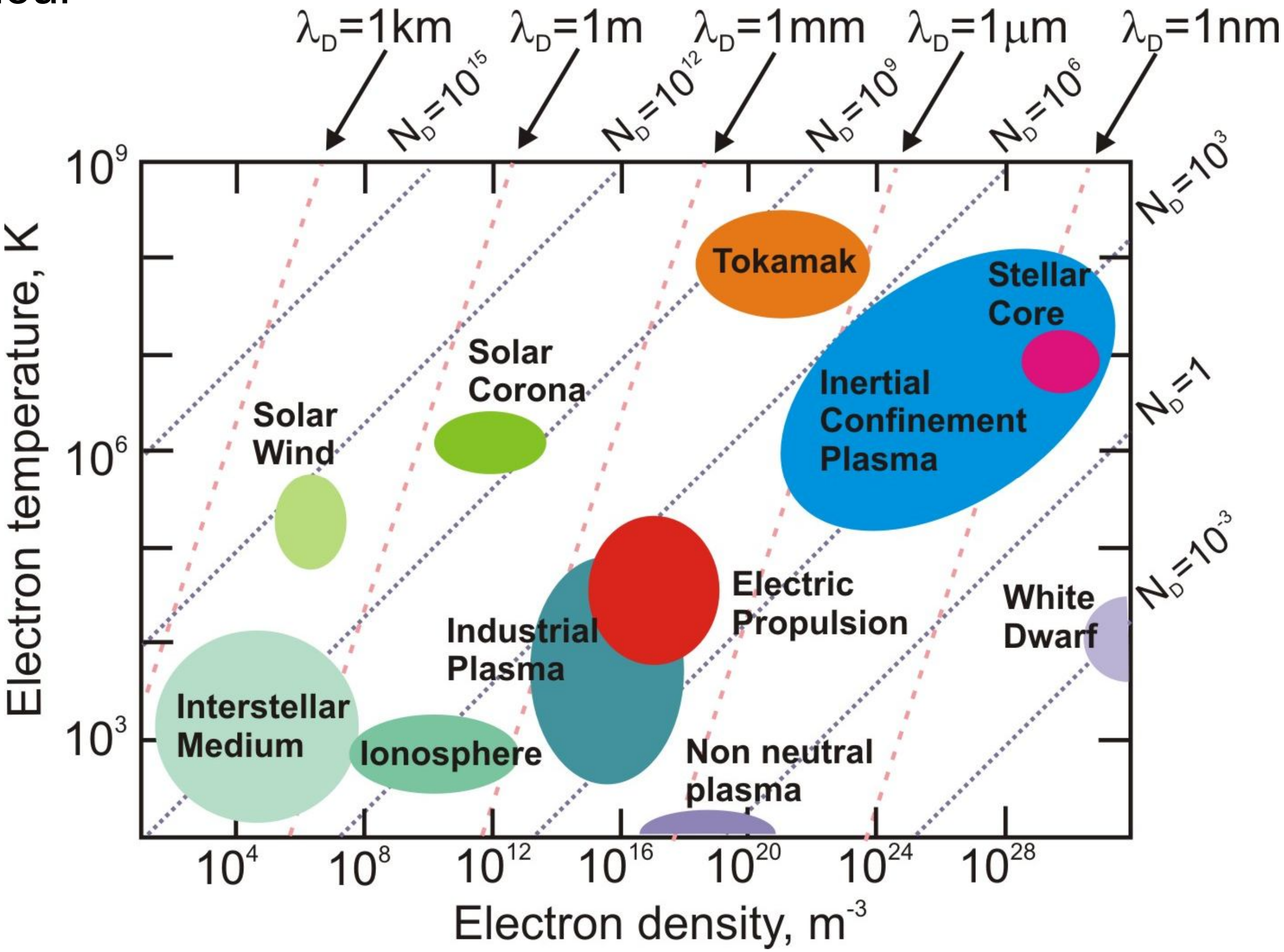
$$\frac{d^2}{dt^2}(\delta x) + \frac{e^2 n_0}{m_s \epsilon_0}(\delta x) = 0 \qquad \omega = \sqrt{\frac{ne^2}{\epsilon_0 m_s}}$$





Plasma Classification

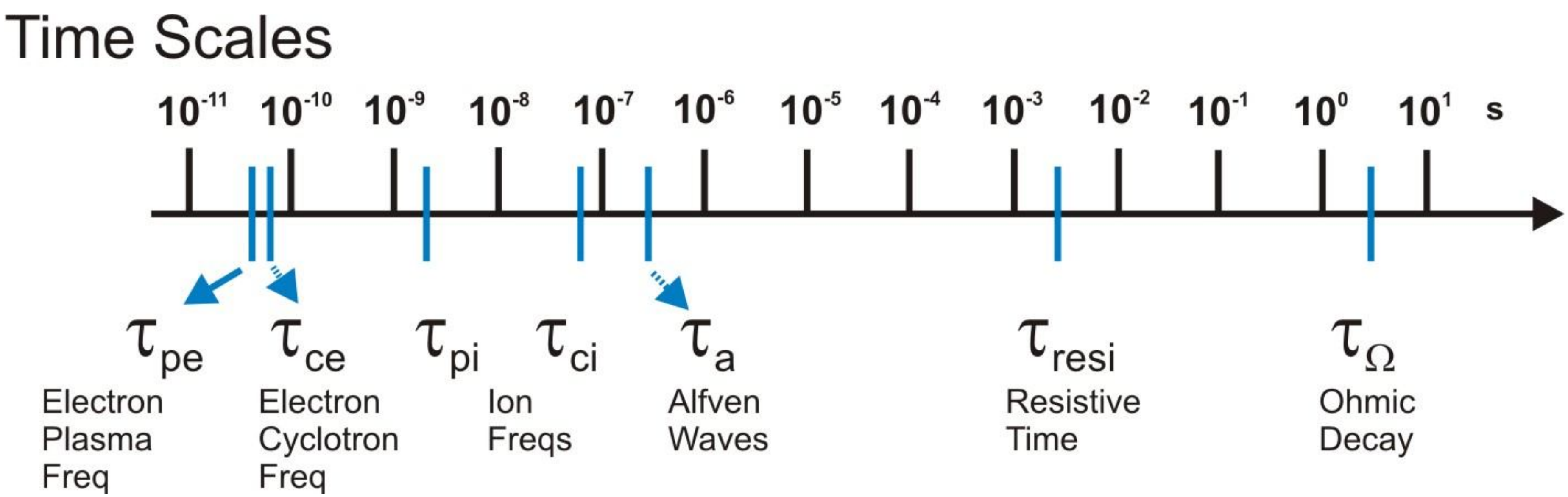
- Different types of classification exists e.g. Cold plasma (non-thermal) – Hot plasma (thermal), Collisional-non collisional, neutral –non neutral, complex
- They are all broadly classified depending on density, temperature and behaviour



Plasma : Time Scales

Different phenomena are observed at different time scale ranging from picosecond to few second scale (some of them even extend to years!)

Consider an example of hydrogen plasma with density 10^{19} m^{-3} , T= 1 keV, inside a Tokamak (L~1 m) with magnetic field of 1 T will yield time scales of plasma period 3.5×10^{-11} s, ion cyclotron period 6.5×10^{-8} s, Alfven wave period 1.4×10^{-7} s, resistive time 2 ms, and Ohmic decay time ~3 s.



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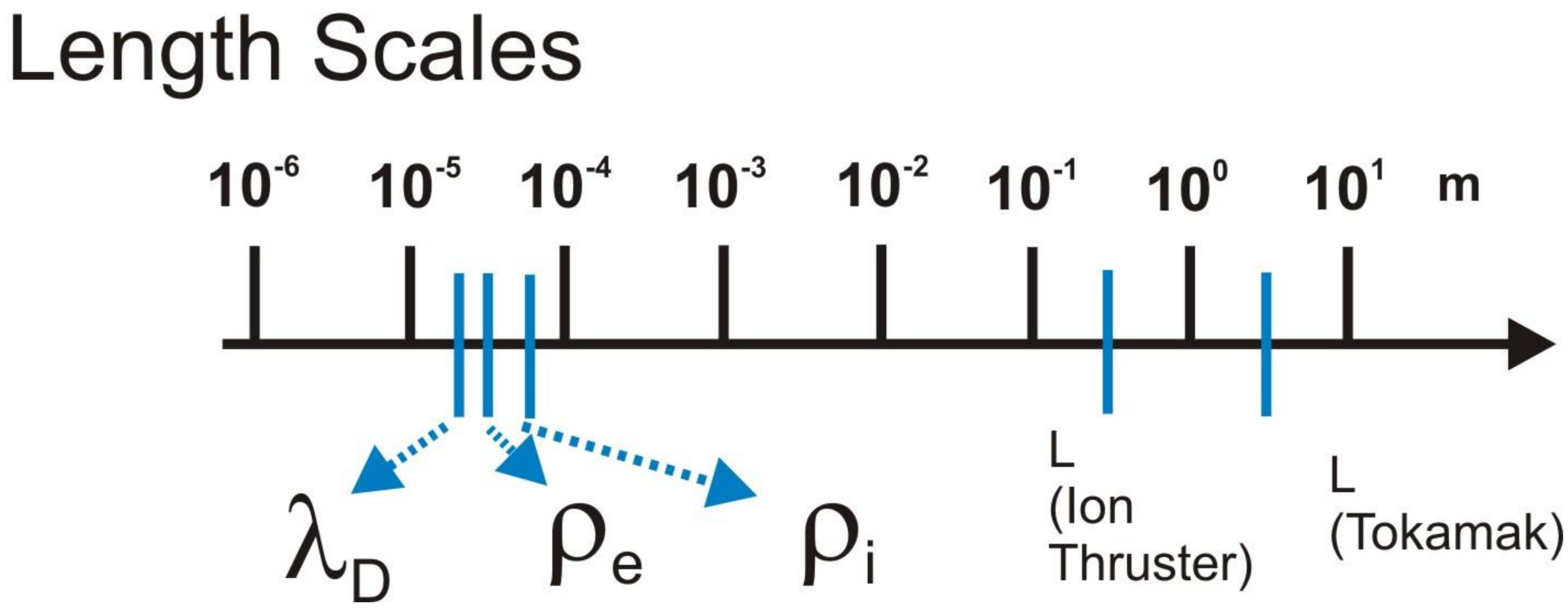
Plasma : Length Scales

Similar to time scale the length scale also vary depending upon system / plasma. For Tokamak typical characteristic length are in order of meter with Debye length in the order of 10^{-5} m and Larmor radius 10^{-6} m, whereas for solar corona characteristic length is km and Debye length is in the order of centimetre.



Plasma Modelling : Hierarchy

3 Level of description Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) Hierarchy

- Exact microscopic description
- Kinetic description
- Fluid description

N –body Model

- Plasma or particle beam is made of a number of particles
- Classical or relativistic dynamics : obeys Newton‘s Law

$$\frac{d\gamma m\mathbf{v}}{dt} = \sum \mathbf{F}_{ext} \qquad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

- Plasma contains more than 10^{10} particles
- Simulation would be too expensive
- Each operation count would introduce numerical error

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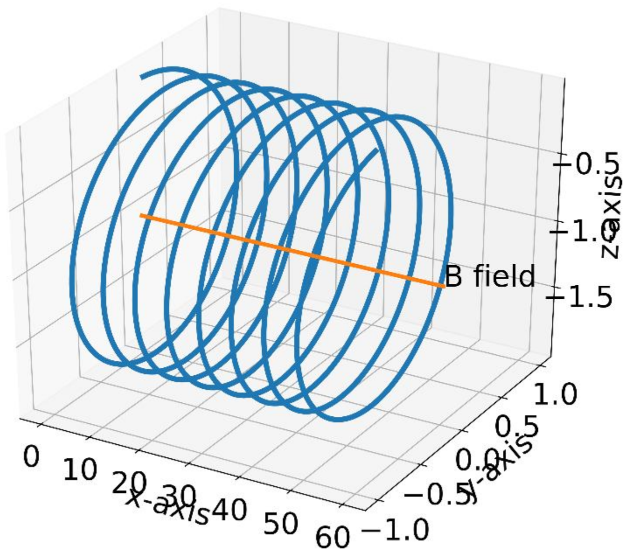
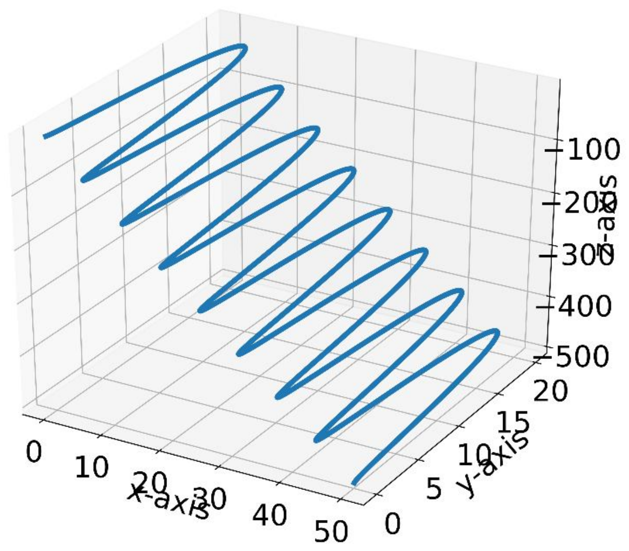
Single Particle Motion

- Motion of single particle for basic understanding of trajectories
- Dominated by drift motions in EM fields : E×B drift, Curvature drifts in Tokamak, gradient B drift for non homogenios fields, drifts due to time varying fields

Eg: charged particle in magnetic field exhibits gyratory motion when additional electric field applied undergoes E×B drift

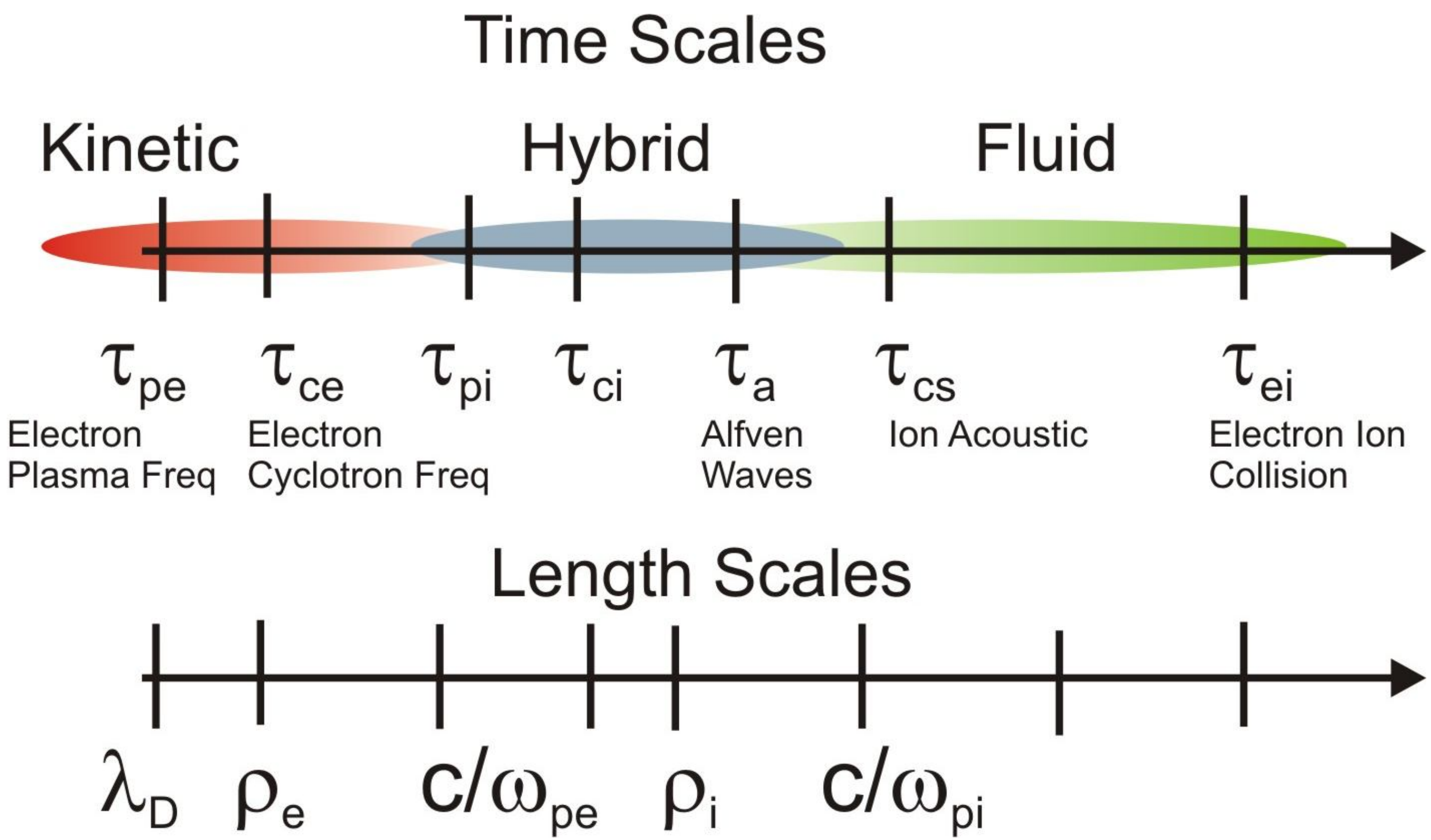
$$\omega_c = \frac{qB}{m} \qquad \rho_L = \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{qB} \qquad \mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$





Kinetic and Fluid Models

- The choice of plasma description, either Kinetic or Fluid model is made based on the temporal and spatial scale
- Hybrid models also employed where one of the plasma component is treated as fluid and other kinetic



Kinetic Model

- Each particle species ***s*** in plasma is characterized by distribution function $f_s(\mathbf{x},\mathbf{v},t)$ corresponding to statistical mean repartition in phase space

- Collisionless Boltzmann equation is given as

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

- Inserting Lorenz force term we get Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

- The collisions can be included in Vlasov equation as

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

- The RHS term in simplest form given as

f_m is distribution function, τ is collision time

$$\left(\frac{\partial f}{\partial t}\right)_c = -\left(\frac{f - f_m}{\tau}\right)$$

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Kinetic Model : Particle Method

Vlasov Maxwell system

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= -\frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial B}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial E}{\partial t} \right) \end{aligned}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\rho(\mathbf{x}) = \sum_j q_j \delta(\mathbf{x} - \mathbf{x}_j)$$

$$\mathbf{J}(\mathbf{x}) = \sum_j q_j \mathbf{v}_j \delta(\mathbf{x} - \mathbf{x}_j)$$

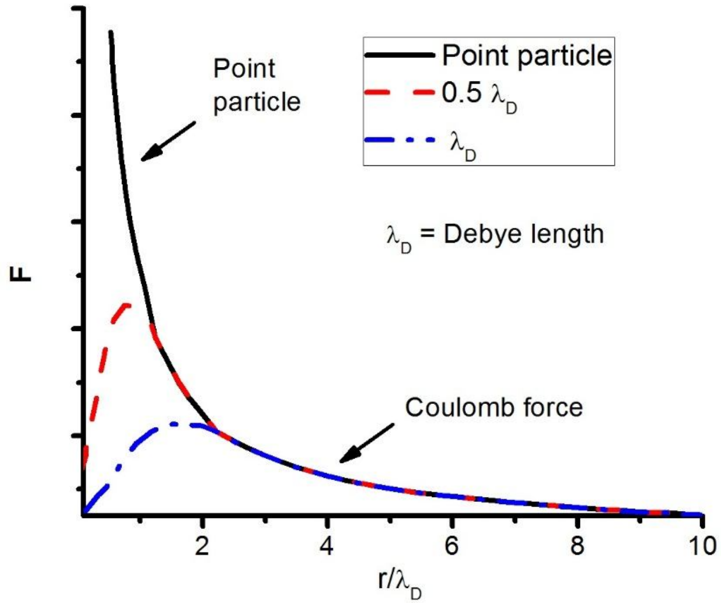
Electric and magnetic fields can be self induced or external from electrodes

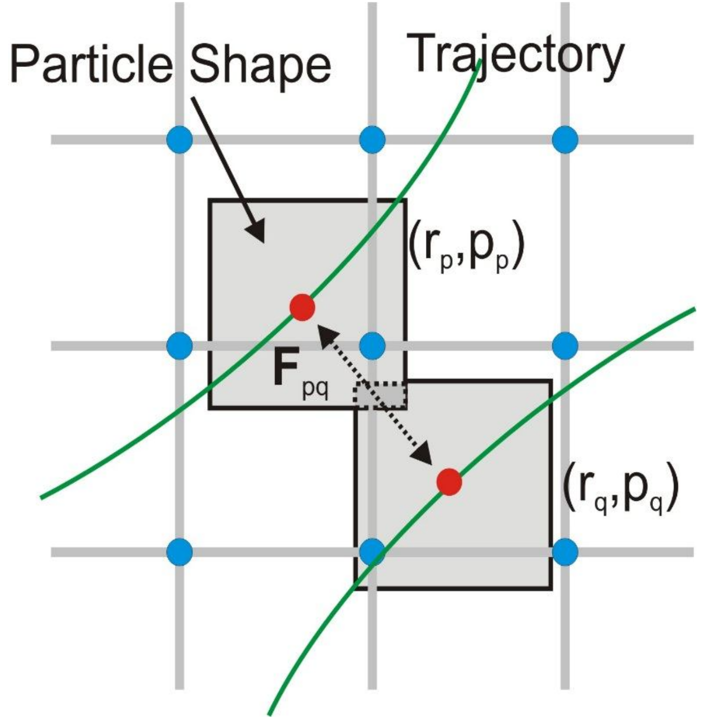
Direct solution will be very expensive

Particle in Cell : Concept

- 6D Vlasov Equations are not practical
- (Re-) intoduce computational particle called super-particle discretize f(**x,v**,t)
- Thus particles in Lagrangian frame are tracked in continuous phase space, and fields are computed on Eulerian mesh

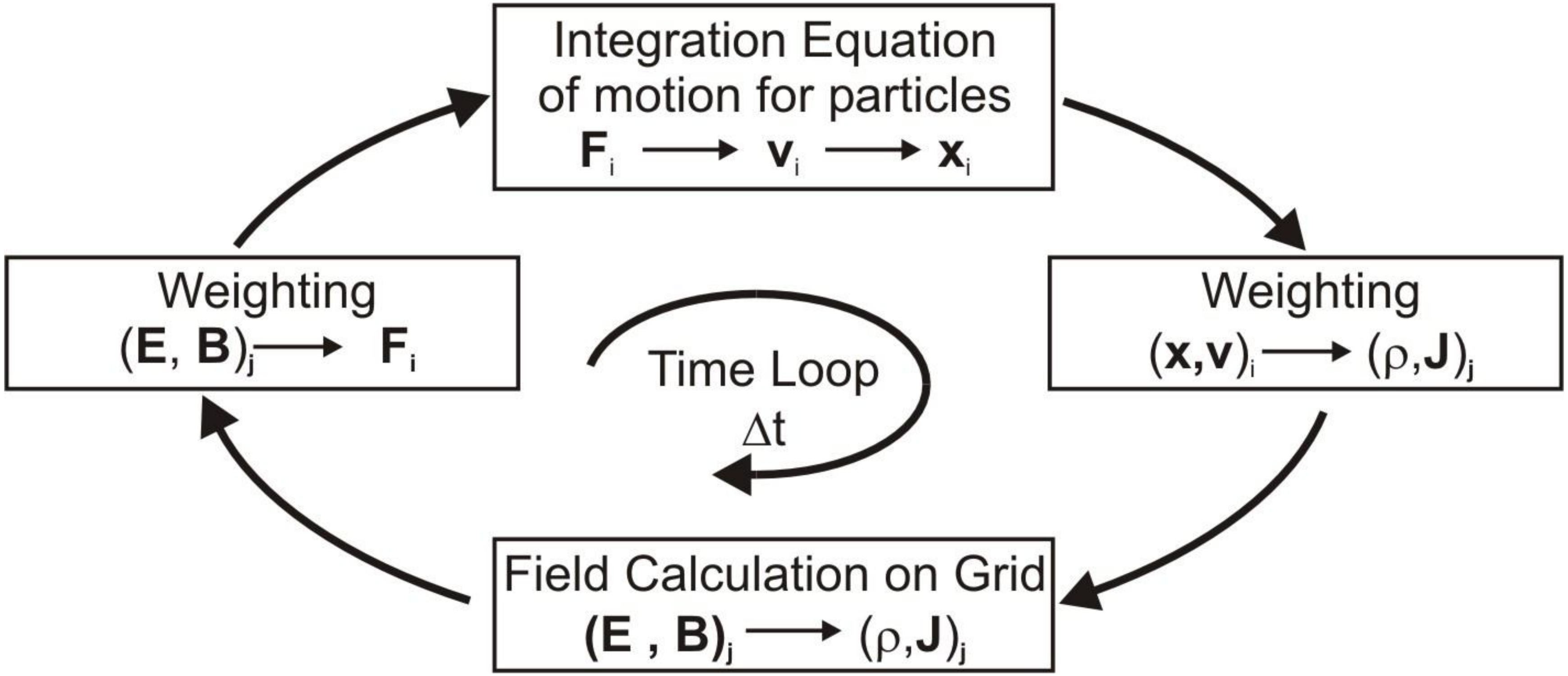
Advantage of finite-size particle is that being finite-sized they interact more weakly than point particle. Thus singularity is avoided as they approach zero distance.





Particle in Cell: The main loop

- Integration of the equation of motion
- Interpolation of charge and current source terms to the field mesh
- Computation of field on grid points
- Interpolation of the fields from mesh to the particle location



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PIC: Mover / Particle Pusher

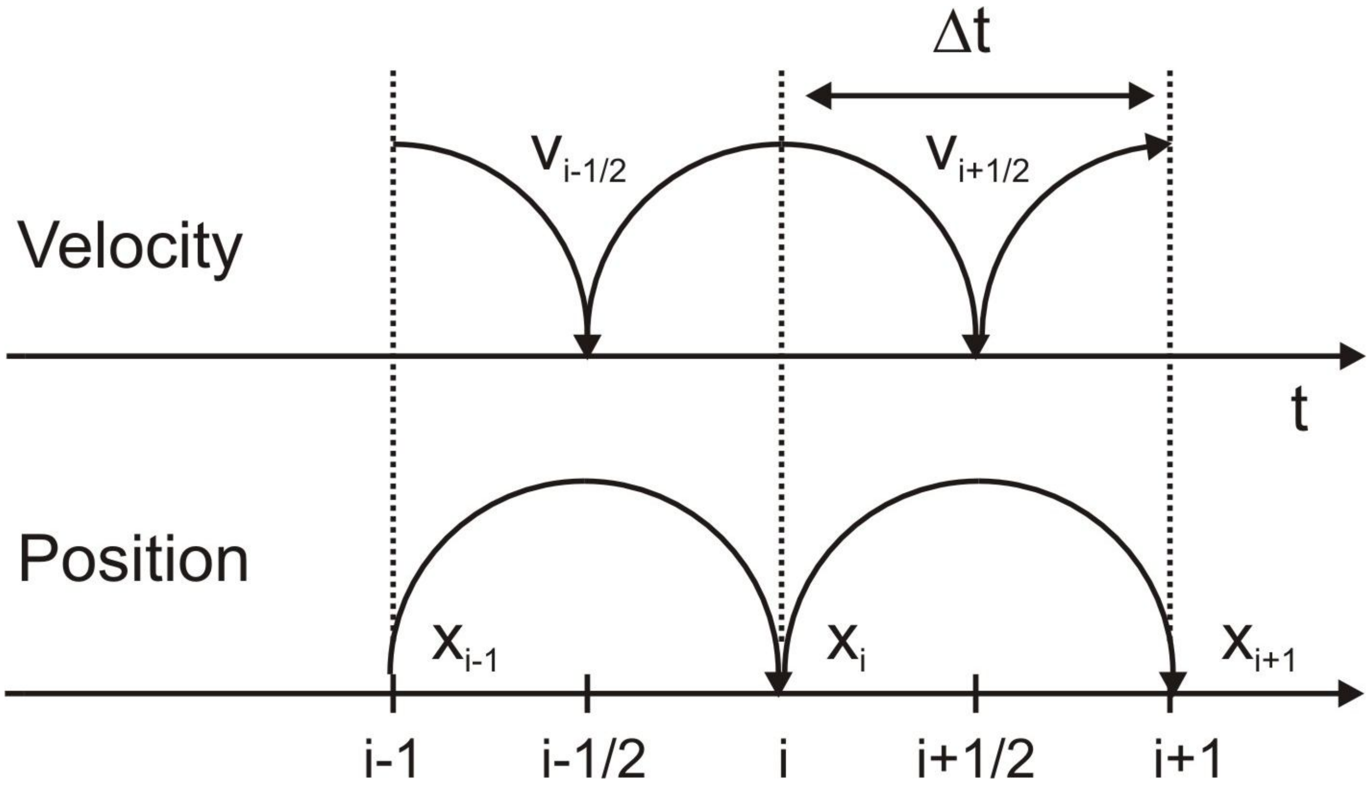
- There are two types of solvers for particle mover Implicit and Explicit
- Explicit method calculate later state of system from current while the implicit method solving the equation involving both the states

Explicit:

$$Y(t + \Delta t) = F(Y(t))$$

Implicit:

$$G\left(Y(t), Y(t + \Delta t)\right) = 0$$



Explicit methods : Leapfrog method

First center difference form was proposed by Buneman

$$\frac{\mathbf{v}_{i+1/2} - \mathbf{v}_{i-1/2}}{\Delta t} = \frac{q}{m}\left[\mathbf{E}_i + \frac{\mathbf{v}_{i+1/2} - \mathbf{v}_{i-1/2}}{2} \times \mathbf{B}_i\right]$$

The scalar components are solved simultaneously

Boris Method : Equations for E and B are separated within half time period

$$\mathbf{v}_{i-1/2} = \mathbf{v}^- - \frac{q\mathbf{E}}{m}\frac{\Delta t}{2}$$

$$\mathbf{v}_{i+1/2} = \mathbf{v}^- + \frac{q\mathbf{E}}{m}\frac{\Delta t}{2}$$

$$\frac{\mathbf{v}^+ - \mathbf{v}^-}{\Delta t} = \frac{q}{2m}\left(\mathbf{v}^+ + \mathbf{v}^-\right) \times \mathbf{B}$$

Relativistic formula involves more steps to accomodate γ changing during jump

PIC: Weighting Methods

Particle in Cell = divide particle quantities like charge in the cell which it is located

Many methods exist, grouped into 4

- Nearest Grid Point
- Cloud in Cell (CIC)
  - Shaped particles
  - PIC methods, linearly shaped particles
- Multiple
  - Dipole, subtracted dipole, etc.
- Higher order methods
  - Splines
  - K-space cutoff in discrete transform

- Possible hybrid methods also exist.

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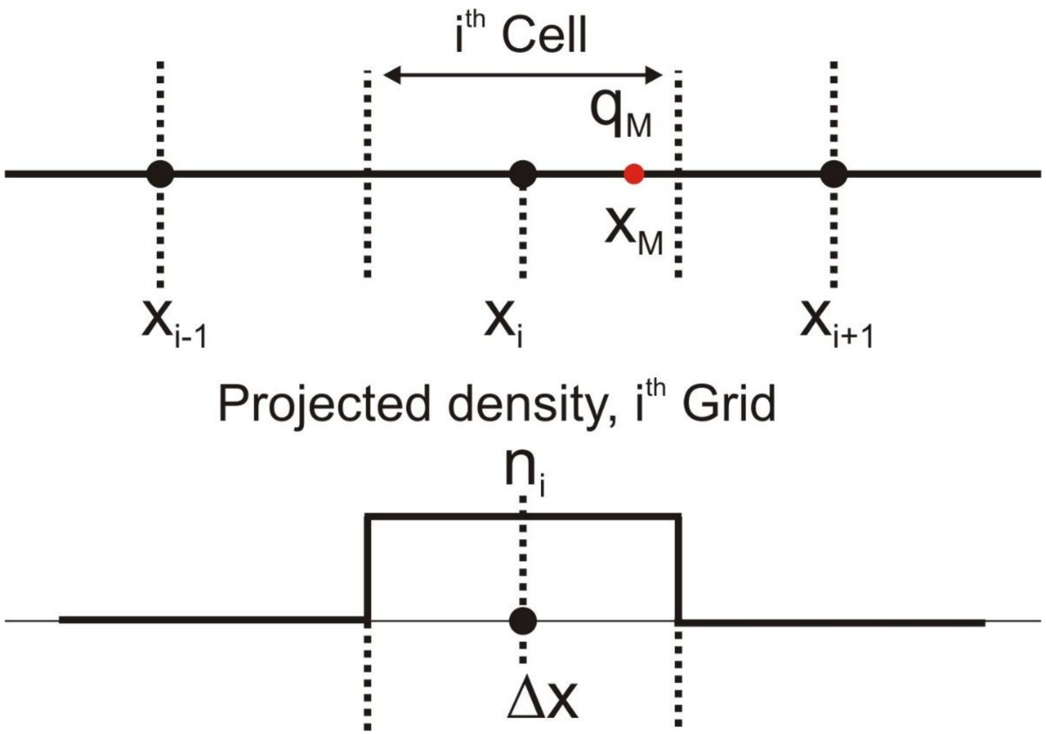
Weighting: Nearest Grid Point

- Nearest Grid Point assigns charges to nearest grid cell
- Fast and easy to implement in 1D, 2D and 3D
- Noisy
- Deposit macro particle charge
- Similary the current densities can also be deposited $q_M \mathbf{v}_M$



- Charge deposition at ith grid point

$$q_i = q_M$$

- Field interpolation to Particle

$$\mathbf{E}x\,|_{x=xM} = E_{xi}$$

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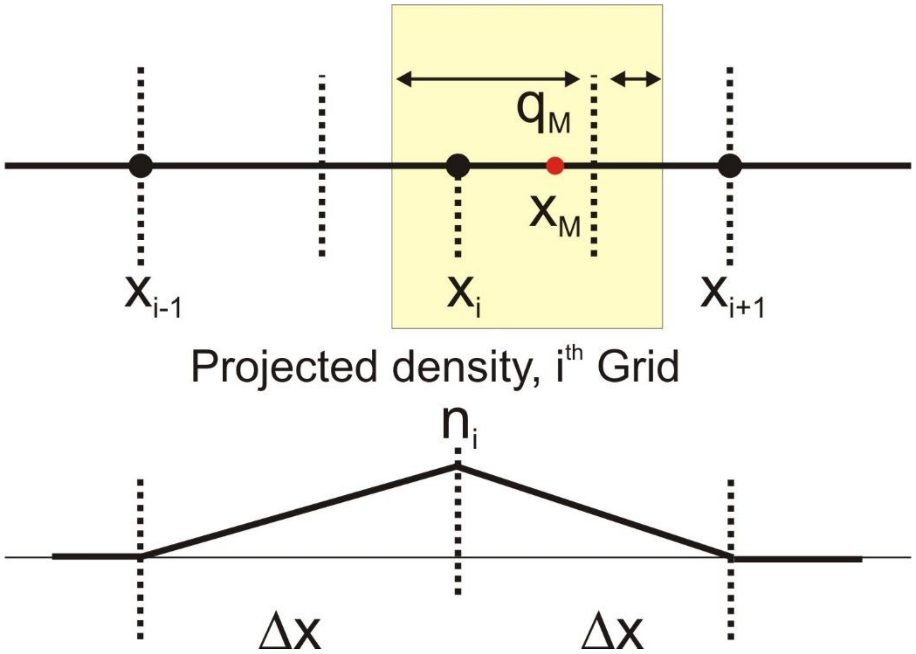
Weighting: Cloud in Cell

- Cloud in cell is smoother than NGP at the cost of additional computation
- Linear interpolation is used results in triangular shaped particles



Charge deposition at grid points

$$q_i = q_M \frac{x_{i+1} - x_M}{\Delta x}$$

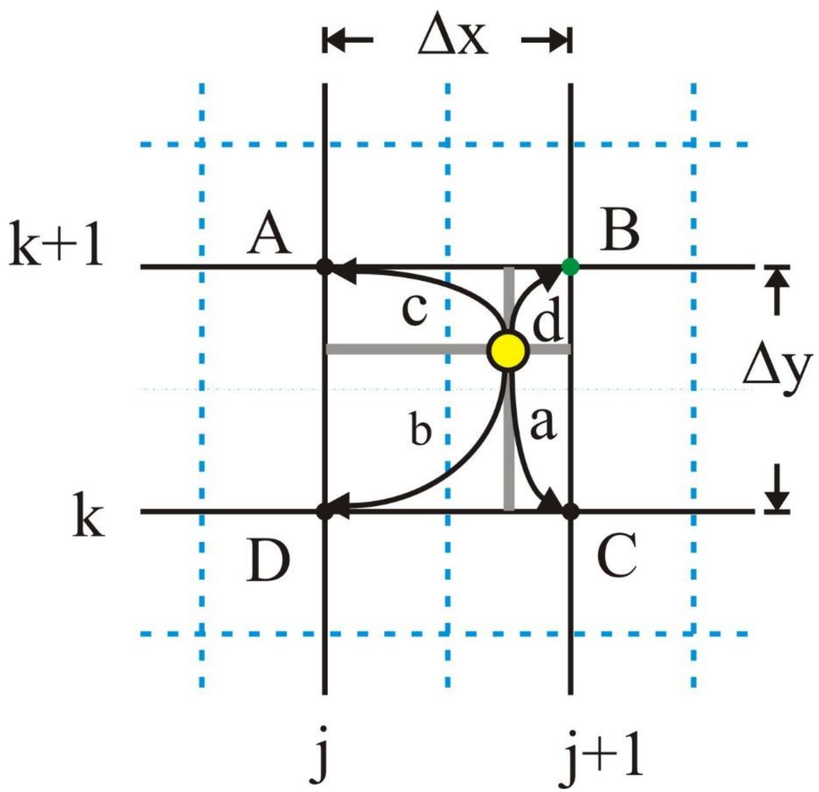
$$q_{i+1} = q_M \frac{x_M - x_i}{\Delta x}$$

Field interpolation to Particle

$$E_x\big|_{x=x_m} = \left[\frac{x_{i+1} - x_M}{\Delta x}\right] E_i + \left[\frac{x_M - x_i}{\Delta x}\right] E_{i+1}$$

Weighting : Charge division in 2D , Area weighting

- In 2D Cloud in Cell method weighting is accomplished using rectangular „area weighting“ to the nearest grid point
- Procedure can be extended in 3D using volume elements
- Currents can be interpolated similar way



PIC: Field calculation on Grid

Most commonly used methods are catagorized into

- Finite Difference Method (FDM)
  - Oldest of the methods
  - Continuous domain is replaced by discrete grid points
  - Derivatives are approximated with differences between neighbouring grid point values
- Finite Element Method (FEM)
  - Continuous domain is divided into elements
  - PDEs are treated as eigenvalue problem and solution is calculated using basis functions
  - This method is more accurate that FDM at the cost higher computational power
- Spectral Element Method
  - In this case also PDEs are transformed into eigenvalue problem but basis functions are higher order
  - Domain can remain continuous
  - This is very recent method though have problems implementing complex geometries

PIC : Electrostatic field solution

The total electric field E is from externally applied field and space charge

$$\mathbf{E} = \mathbf{E_a} + \mathbf{E_s}$$

The Poisson's equation

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

And from Potential we get

$$E = -\nabla \phi$$

Discretized form of Poisson' equation in 1D

$$\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2} = \frac{\rho_i}{\epsilon_0}$$

$$\begin{bmatrix} -2 & 1 & \cdot & \cdot & & & & 0 \\ \cdot & 1 & 2 & 1 & \cdot & \cdot & & \\ \cdot & & 1 & 2 & 1 & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & 1 & -2 & 1 \\ 0 & & & & \cdot & \cdot & 1 & -2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \cdot \\ \cdot \\ \phi_{nx-2} \\ \phi_{nx-1} \end{bmatrix} = \frac{\Delta x^2}{\epsilon_0} \begin{bmatrix} \rho_1 + \frac{\epsilon_0}{\Delta x^2} V_l \\ \rho_2 \\ \rho_3 \\ \cdot \\ \cdot \\ \rho_{nx-2} \\ \rho_{nx-1} + \frac{\epsilon_0}{\Delta x^2} V_r \end{bmatrix}$$

Matrix Eqquation of form

$$A\mathbf{x} = b$$

This can be easly extended in 2D and 3D form

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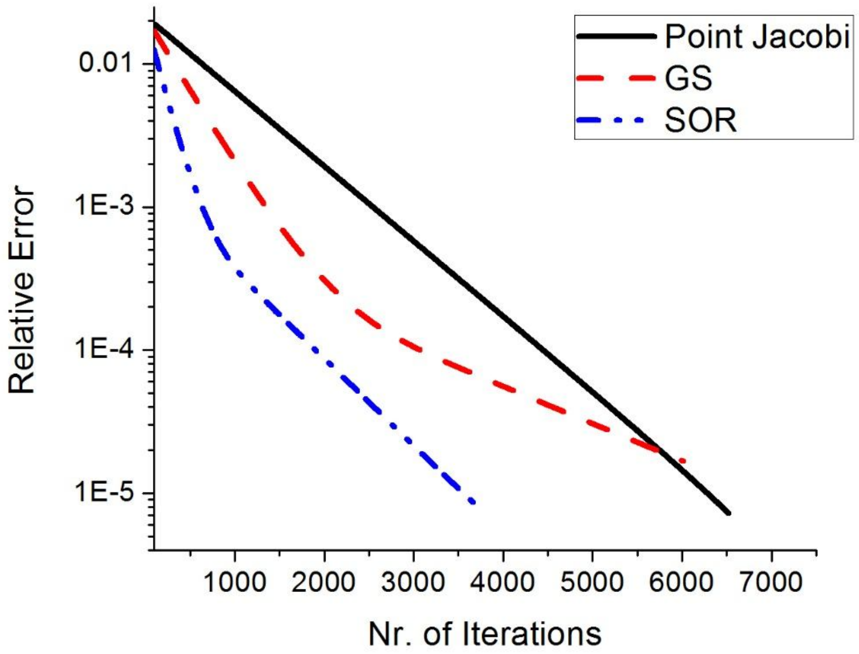
Solution for Matrix Equation

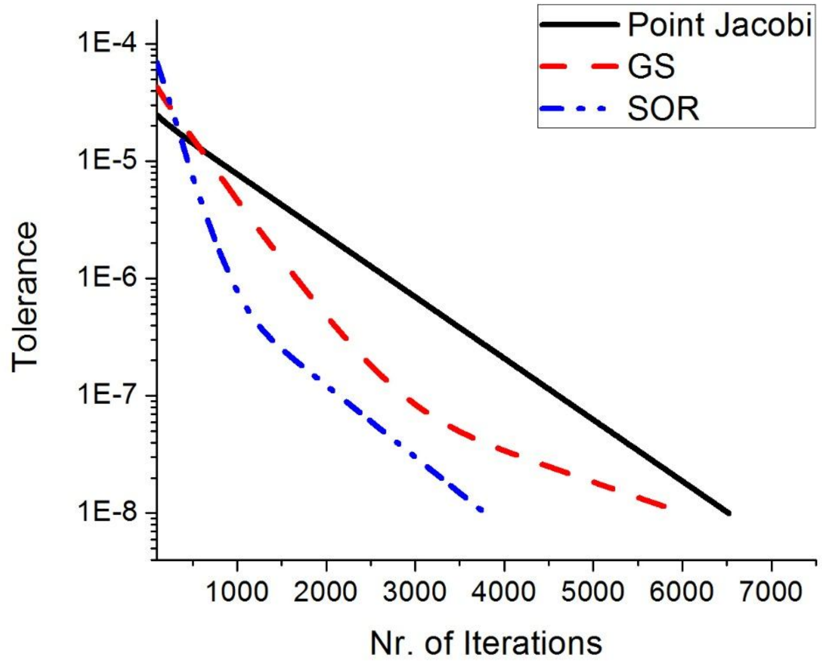
- Direct method
  - Fast inversion of Sparse matrx
  - Computes precise solution to problem
  - Example: Gaussian Elimination
- Spectral method
  - Fast Fourier Transform
  - Periodic boundary conditions can be easily applied
- Iterative methods
  - Most preferred method
  - Starting from Initial guess, in successive approximations solution is achieved
  - Example: Jacobi, Gauss-Seidel, Successive Over Relaxation, Multigrid

Iterative Methods

- Jacobi (Point-Jacobi),
- Gauss-Seidel
- Succesive Over Relation(SOR)

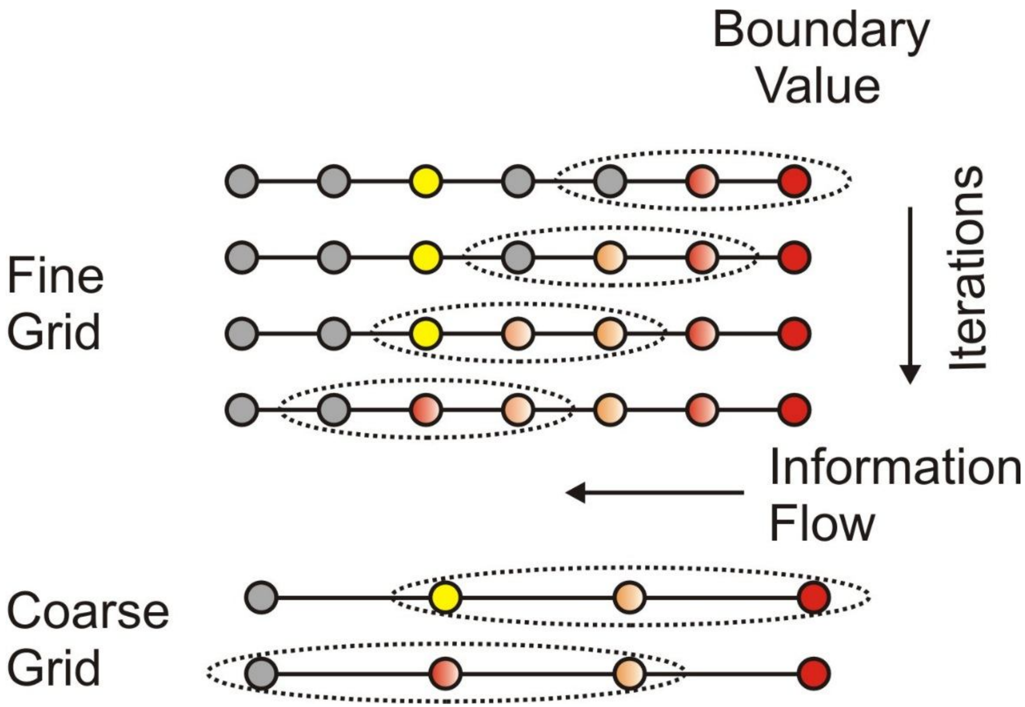
In each iterative step e = ||**A** x - b|| is minimized
In practical example we use tolerance or the change from last step





Multigrid Method

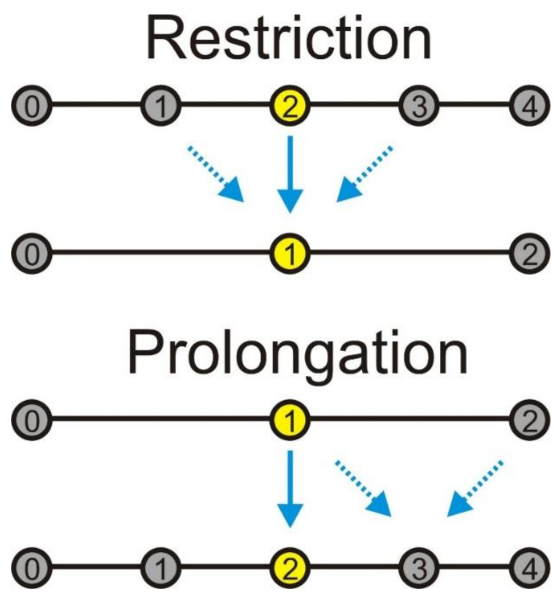
- There are different multigrid strategies
- Main idea is to accelerate convergence of basic iterative method i.e. Relaxation by solving problem on coarser grid time to time.
- The stretching of Finite Difference stencil one can propogate data faster e.g. the boundary values



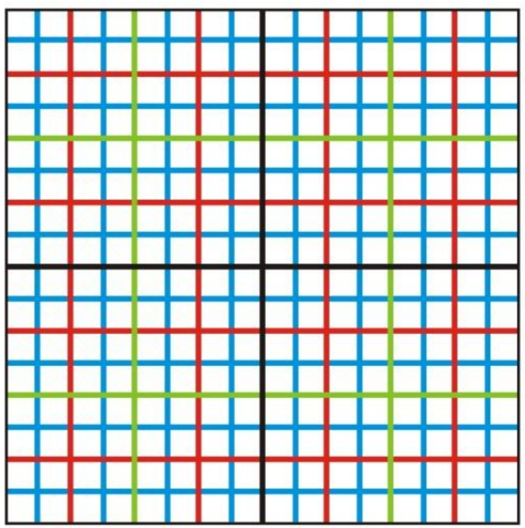
Multigrid Method: V-cycle

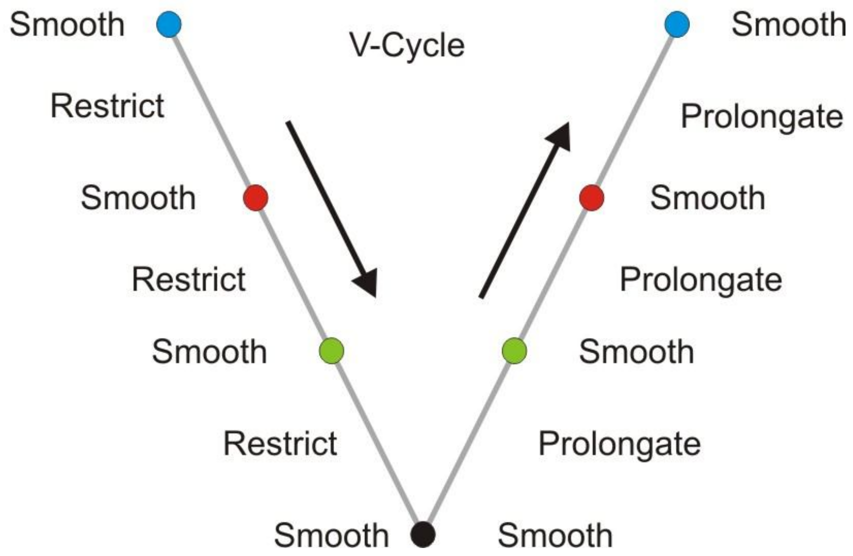
Basic steps involve

- Smoothing: reducing high frequency erros
- Residual comuptation: residual error
- Restriction: downsampling residual error
- Prolongation: interpolation of error to coarser mesh
- Correction : Addition to finer mesh



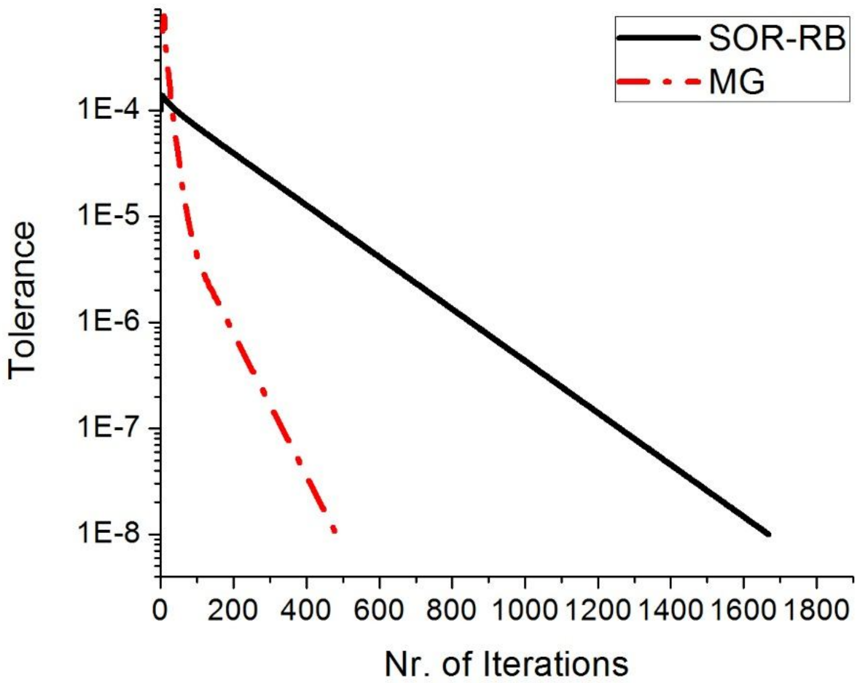
2-D Mesh





Multigrid Method: Comparison

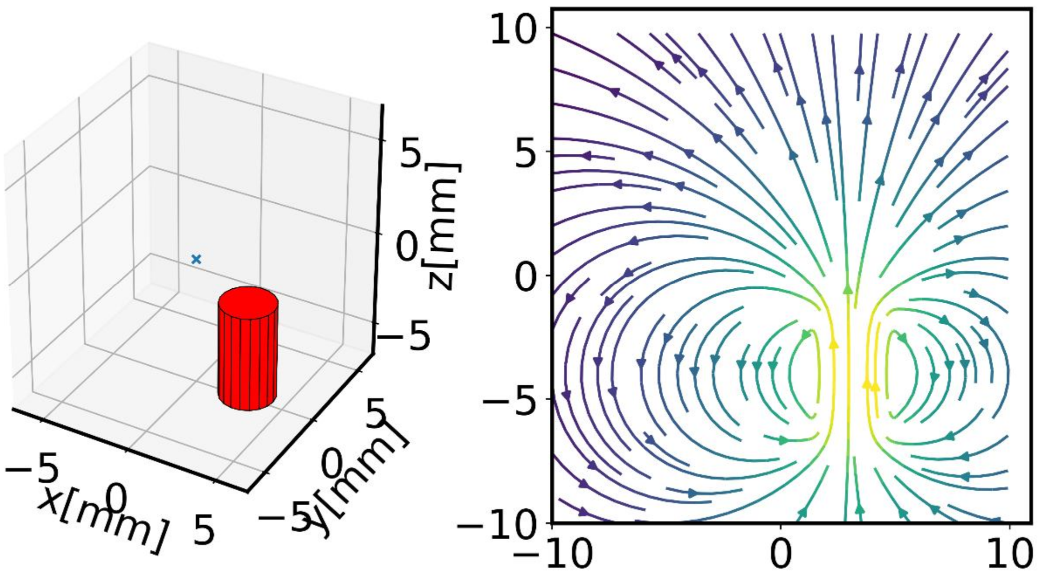
- Other than V-cycle one can have F-cycle or W-cycle
- In this example only 2 levels are used
- SOR algorithm is used for relaxation step



PIC : Magnetic field

- The external magnetic field can have source either current carrying coil or permanent magnets. Magnetic field from current loops can be easily calculated from Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{l} \times \mathbf{r}'}{|\mathbf{r}'|^3}$$



- Magnetic field from Hard (permanent) magnets can be established for simple geometries using analytical formula (Ferrite, NdFeB, similar)
- Soft magnets need extensive database of B-H curve (Hysteresis)
- COMSOL like codes are used in practice

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PIC : Electrodynamic field solution

- The electrodynamic part is solved using inhomogeneous wave equation also known as Telegraphers equation

$$\nabla^2 - \mu_0\epsilon_0 \frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{x},t) = \mu_0 \frac{\partial}{\partial t}\mathbf{j}(\mathbf{x},t)$$

- Assuming harmonically varying fields and currents, we write in complex form,

$$\begin{aligned} \mathbf{E}(\mathbf{x},t) &= \tilde{\mathbf{E}}(\mathbf{x})e^{i\omega t} \\ \mathbf{B}(\mathbf{x},t) &= \tilde{\mathbf{B}}(\mathbf{x})e^{i\omega t} \\ \mathbf{j}(\mathbf{x},t) &= \tilde{\mathbf{j}}(\mathbf{x})e^{i\omega t} \end{aligned}$$

$$\left(\nabla^2 + \mu_0\epsilon_0\omega^2\right)\mathbf{E}(\mathbf{x},t) = i\omega\mathbf{j}(\mathbf{x},t)$$

Here **j** is externally induced currents and from plasma

$$\mathbf{B}(\mathbf{x},t) = \frac{i}{\omega}\nabla \times \mathbf{E}(\mathbf{x},t)$$

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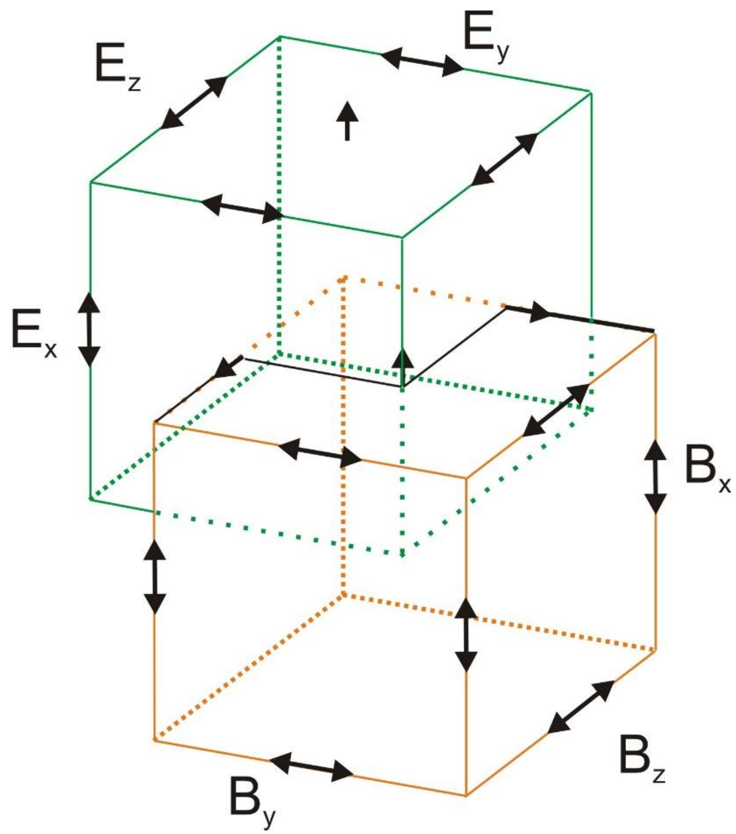
PIC : ElectroMagnetic field solution FDTD

- In Finite difference Time domain staggered space and time stencil call Yee grid is used to solve Maxwell's equation for **E** and **B**
- Current deposion using continuity equation in which case Gauss law is preserved
- In direct current deposit method Boris correction is used additionally which is modified form of Gauss' laws



$$\frac{B_{y\,k+1/2}^{\;n+1/2} - B_{y\,k+1/2}^{\;n-1/2}}{\Delta t} = -\frac{E_{x\,k+1}^{\;n} - E_{x\,k}^{\;n}}{\Delta z}$$

$$\frac{E_{x\,k}^{\;n+1} - E_{x\,k}^{\;n}}{\Delta t} = -c^2\left(\frac{B_{y\,k+1/2}^{\;n+1/2} - B_{y\,k-1/2}^{\;n+1/2}}{\Delta z}\right) - \mu_0 c^2 J_{x\,k}^{\;n+1/2}$$

Particle in Cell : Constraints

Two important conditions to be satisfied

$$\omega_0 \Delta t < 2$$
$$\Delta x \leq \lambda_D$$

Third condition $N_D >> 1$ holds true for plasmas except strongly coupled plasmas

According to Tskhakaya et al, and Hockeney and Eastwood modified to

$$\omega_{pe} \Delta t \leq 0.2$$
$$\Delta x < 3.4 \lambda_D$$

Example: Consider plasma with density 10^{16} m^{-3} and temperature 10 eV

This gives Debye length $\lambda_D = 2.35\times10^{-4}$ m and plasma frequency = 5.64×10^{9} s^{-1}

Hence we have

$\Delta t \leq 3.55 \times 10^{-11}$ s and $\Delta x < 8\times10^{-4}$ m

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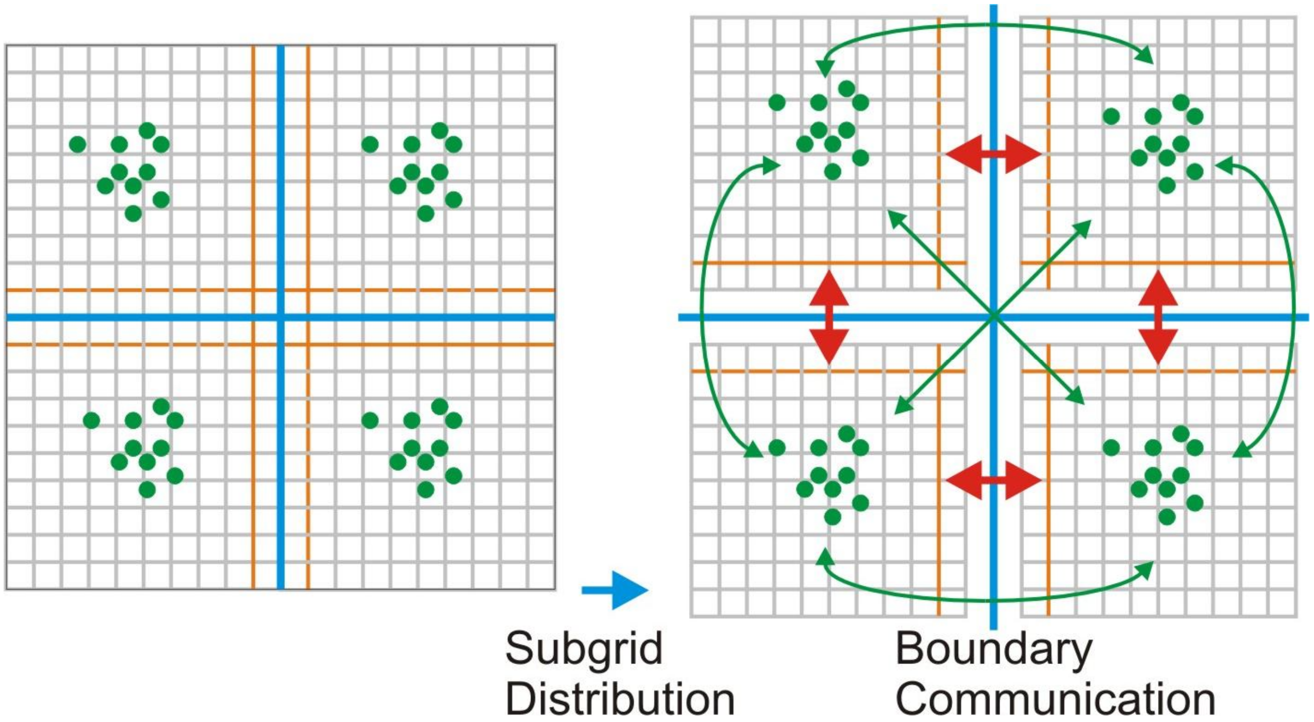
Parallelization and Domain Decomposition

- Parallel architecture instead serial instruction , Divide and conquer
- Subdivision of domain among available CPUs communicatin using MPI
- Shared memory → parallel work load within core using OpenMP
- Communication of boundaries (Fields only straight neighbours, particles overall)



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Particle in Cell : Collisions

- Monte Carlo Collision method can be used
- In each time step probability for collision is calculated for each particle, depending on target density (n_T), velocity of particle (v) and energy dependent cross section (σ_T).

$$P_i = 1 - \ exp(-\Delta t v \sigma_T(E_i) n_T(\mathbf{x}_i))$$

- To reduce computational efforts maximum collision frequency is introduced

$$\nu_{max} = max(n_T(\mathbf{x}_i)) max(\sigma_T(E)) max(v)$$

- Thus maximum number of partilces to be collided can be determined

$$N_{max} = N(1 - exp(-\nu_{max} \Delta t))$$

- Different type of collisions can be simulated : electron-neutral, ion-neutral, etc.

Particle in Cell : Direct Simulation Monte Carlo

- Method for simulating inter particle collision; Bird‘s method
- For neutral gas simulation, analogue to PIC method for neutral gases
- Advantage over CFD is that it does not assume any distribution function
- Probability of collision in the cell is given in terms of macroparticle weight (F_N), total cross section (σ_T), relative velocity (c_r), and cell volume (V_c)

$$P = F_N \left(\frac{\sigma_T c_r \Delta t}{V_c} \right)$$

- The random number is used to determine colliding pairs
- No Time Counter (NTC) is used or reduced computational time

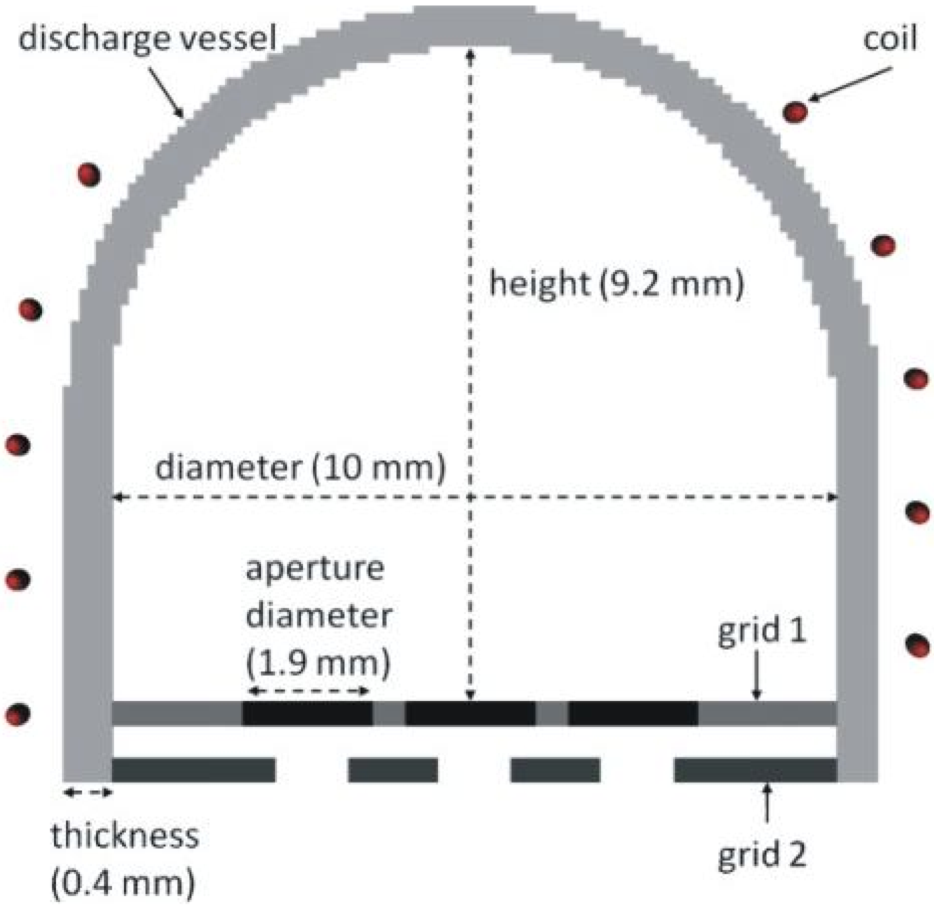
Particle in Cell : Ion Thruster

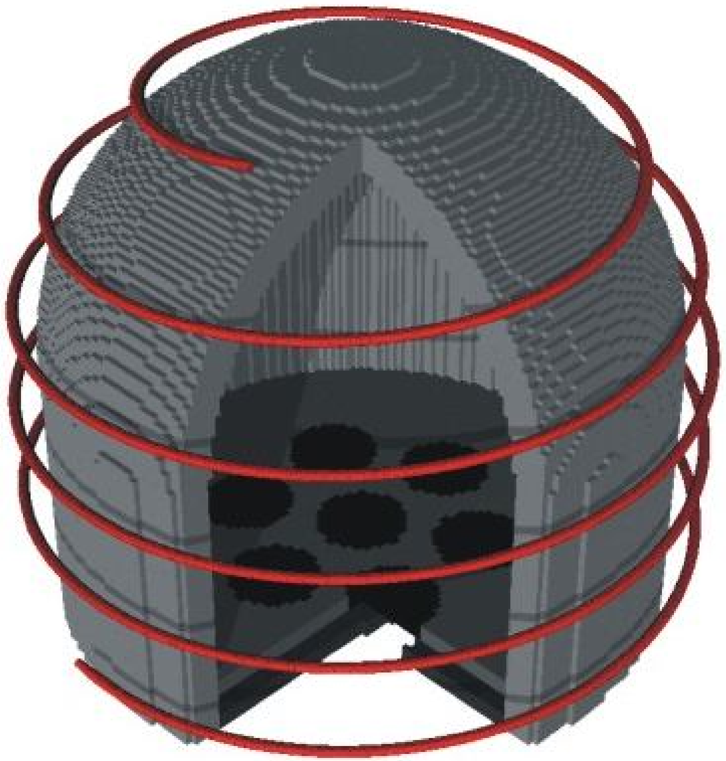
- Radio Frequency Ion Thruster
- Electrodynamic plasma description
- Used for ultra fine positioning

Dimension:

RIT1.0 has diameter of 1 cm with dome shape

Induction coil : 5 loops, r=6 mm





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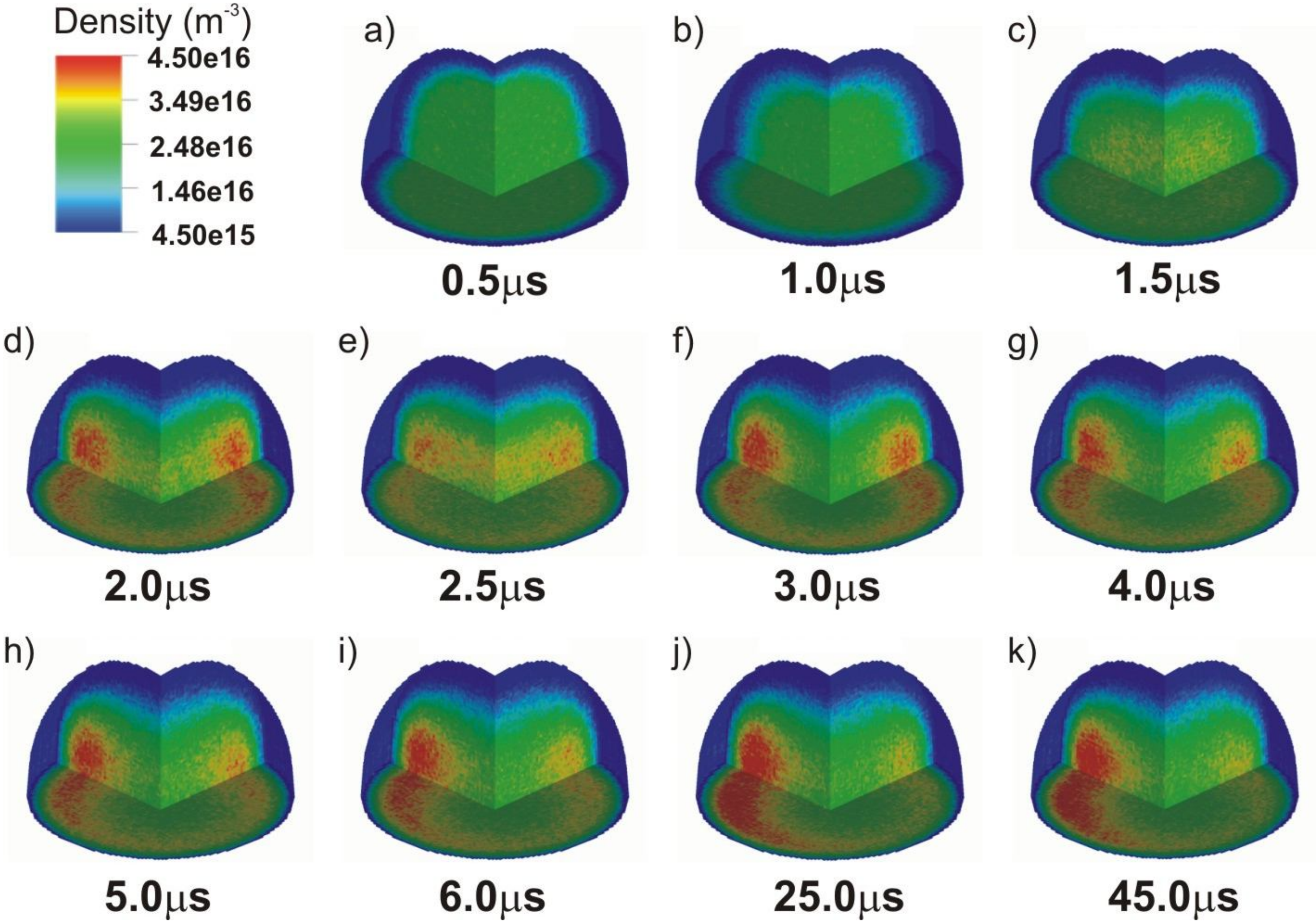
Particle in Cell : Ion Thruster

Initial condition: homogeneous ion and electron densities $2.5\times10^{16}/m^3$

Cell size 0.1 mm, t =5×10^{-12} s, 100×100×100 cells for T = 45 µs



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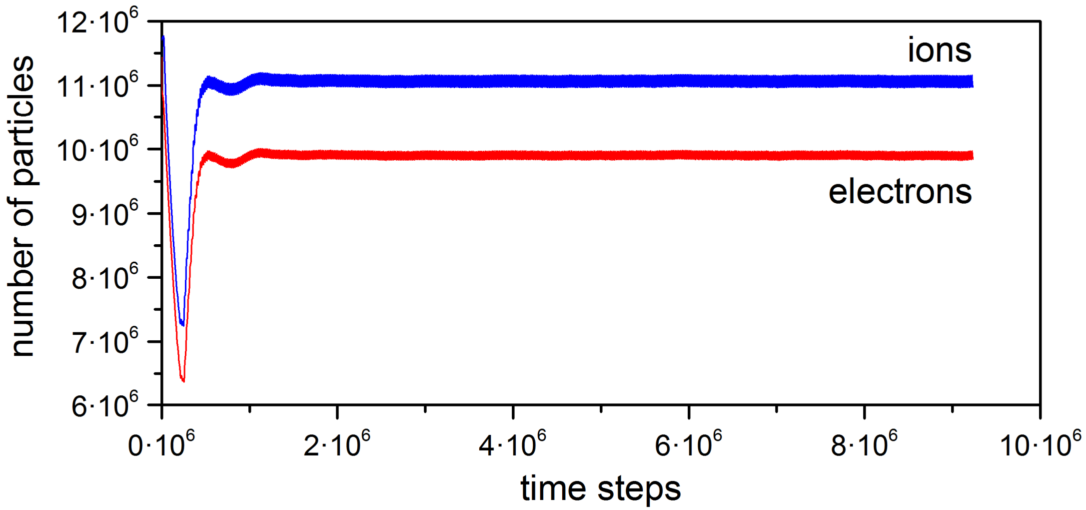
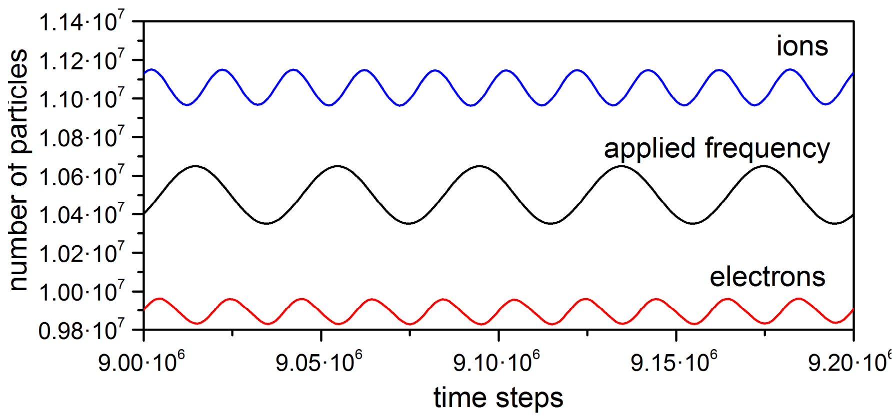
Particle in Cell : Ion Thruster

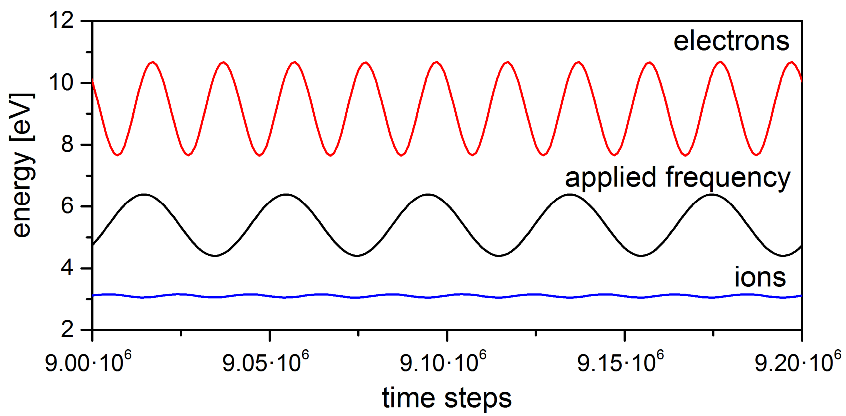
Temporal evolution of charged particles in plasma



Number of charged particles in plasma after stbility



Electron and ion temperature in plasma after stbility



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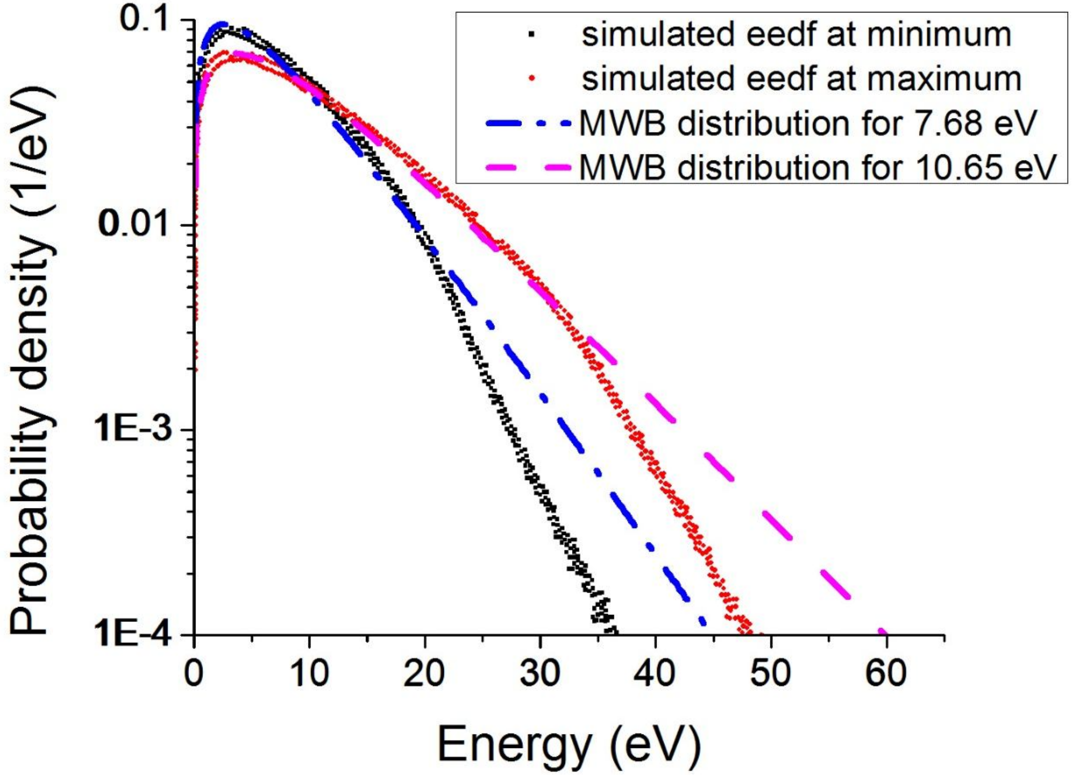
Particle in Cell: Ion thruster

The probability density function shows non Maxwellian distribution after stable plasma conditions



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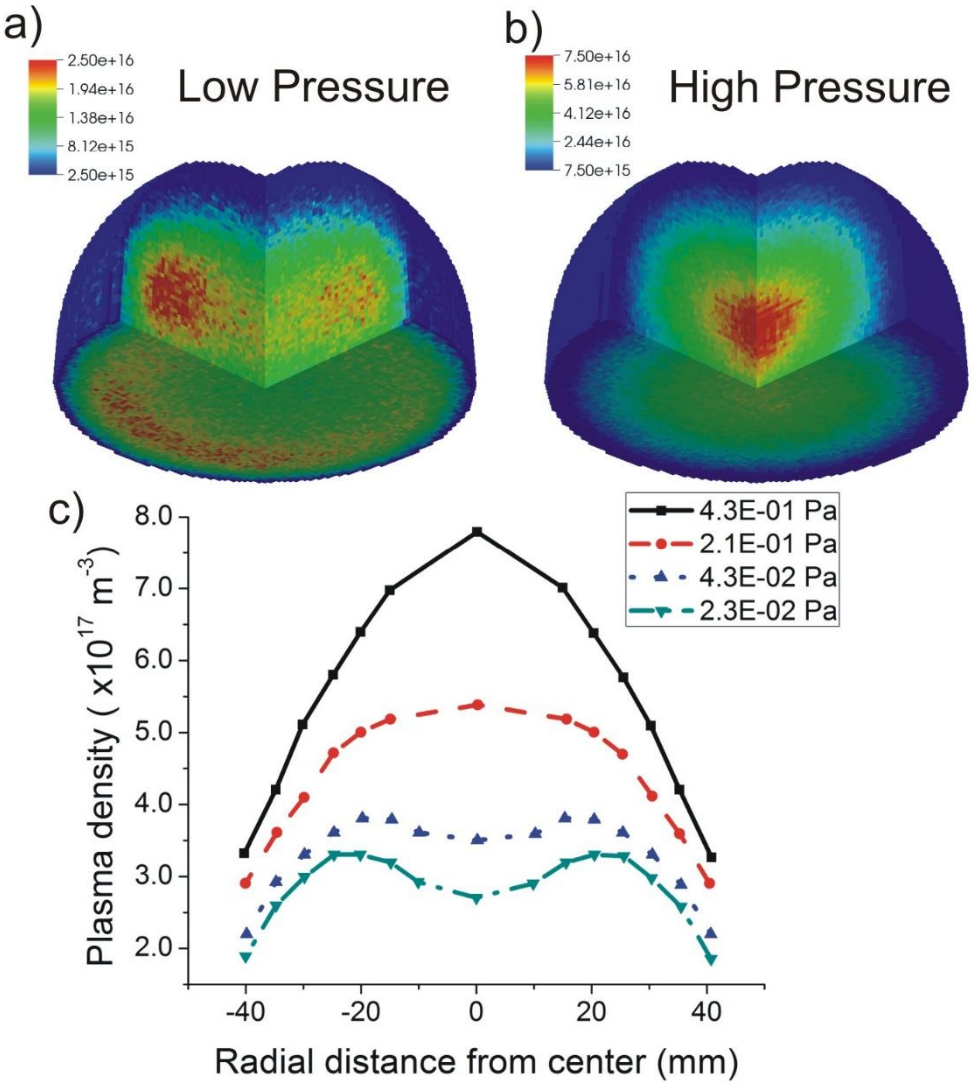
Particle in Cell : Ion Thruster

Comparison with experimental data



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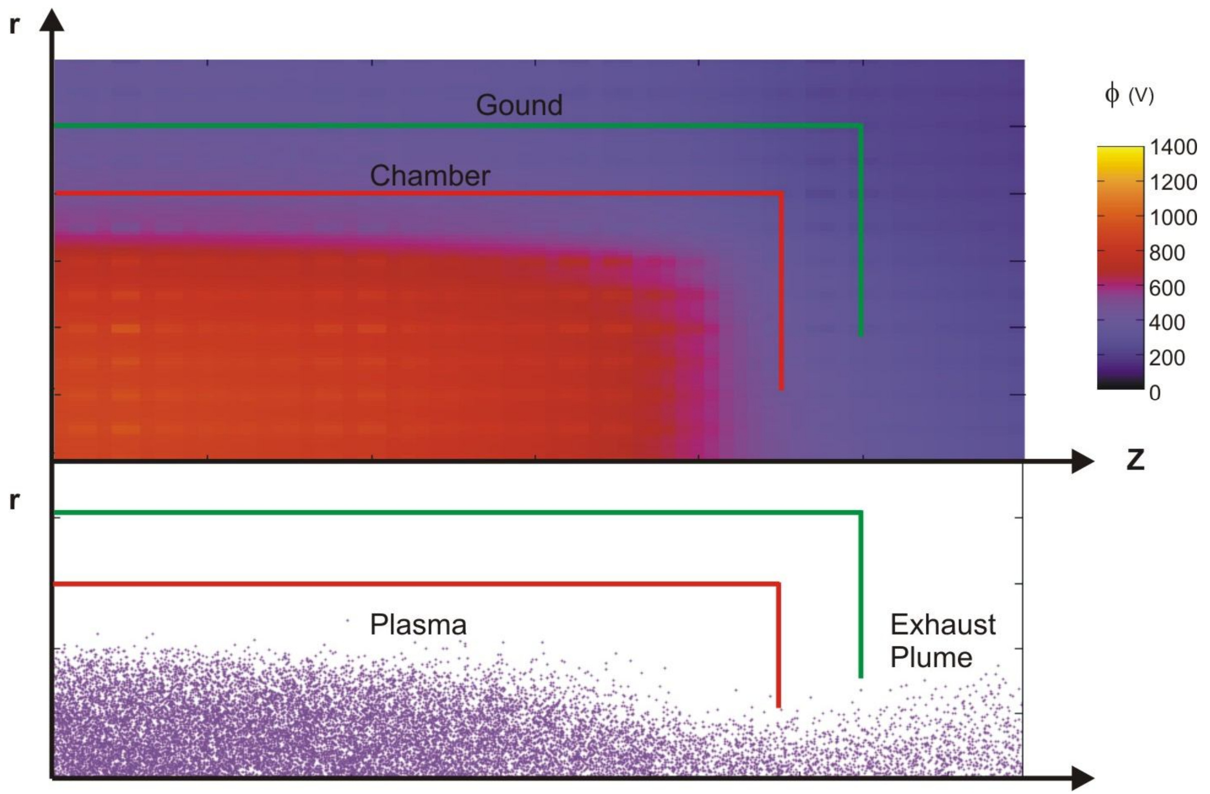
Particle in Cell: r-z symmetry

Reduce computational effort with symmetry argument - Cylindrical symmetry

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

$$\phi_{ij} = \left[\frac{\rho}{\epsilon_0} + \frac{\phi_{i+1}+\phi_{i-1}}{\Delta r^2} + \frac{1}{r_{ij}}\frac{\phi_{i+1}-\phi_{i-1}}{2\Delta r} + \frac{\phi_{j+1}+\phi_{j-1}}{\Delta z^2}\right] \Big/ \left(\frac{2}{\Delta r^2} + \frac{2}{\Delta z^2}\right)$$



Example: Ion thruster with plasma chamber (1.5 kV) and ground electrode forming exhaust plume

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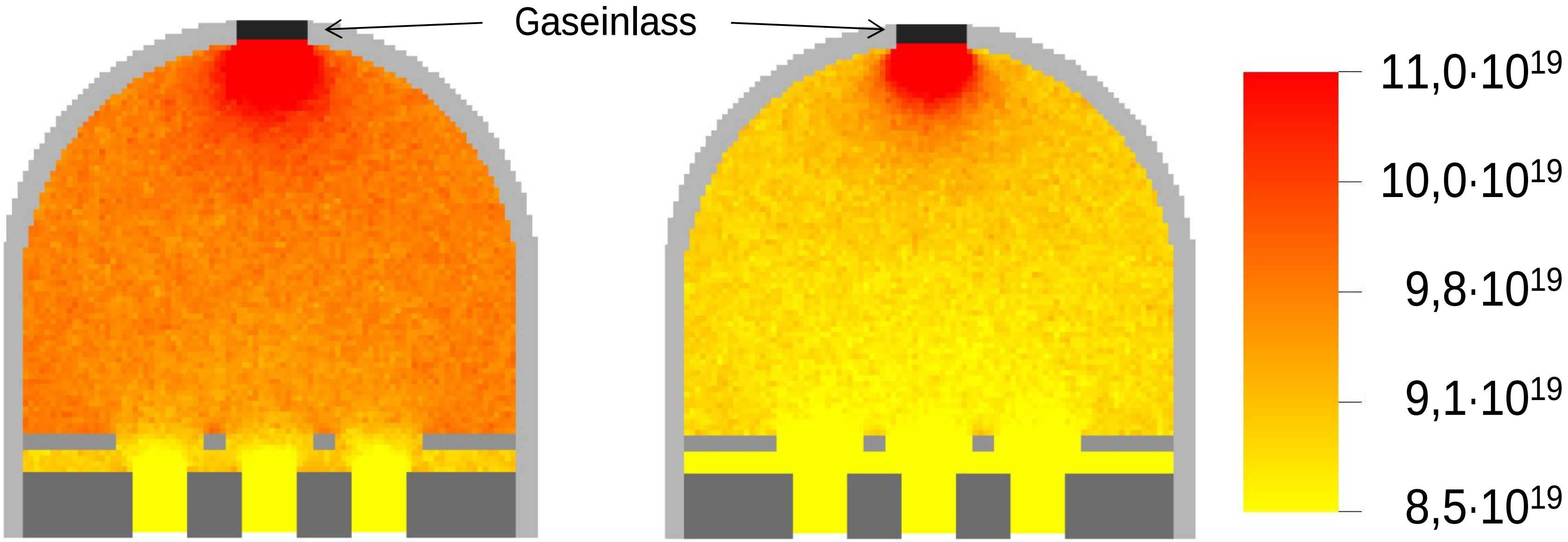
DSMC Simulations of Ion Thruster

Simulation with neutral gas

Neutral gas density in 1/m³



no Plasma

with Plasma

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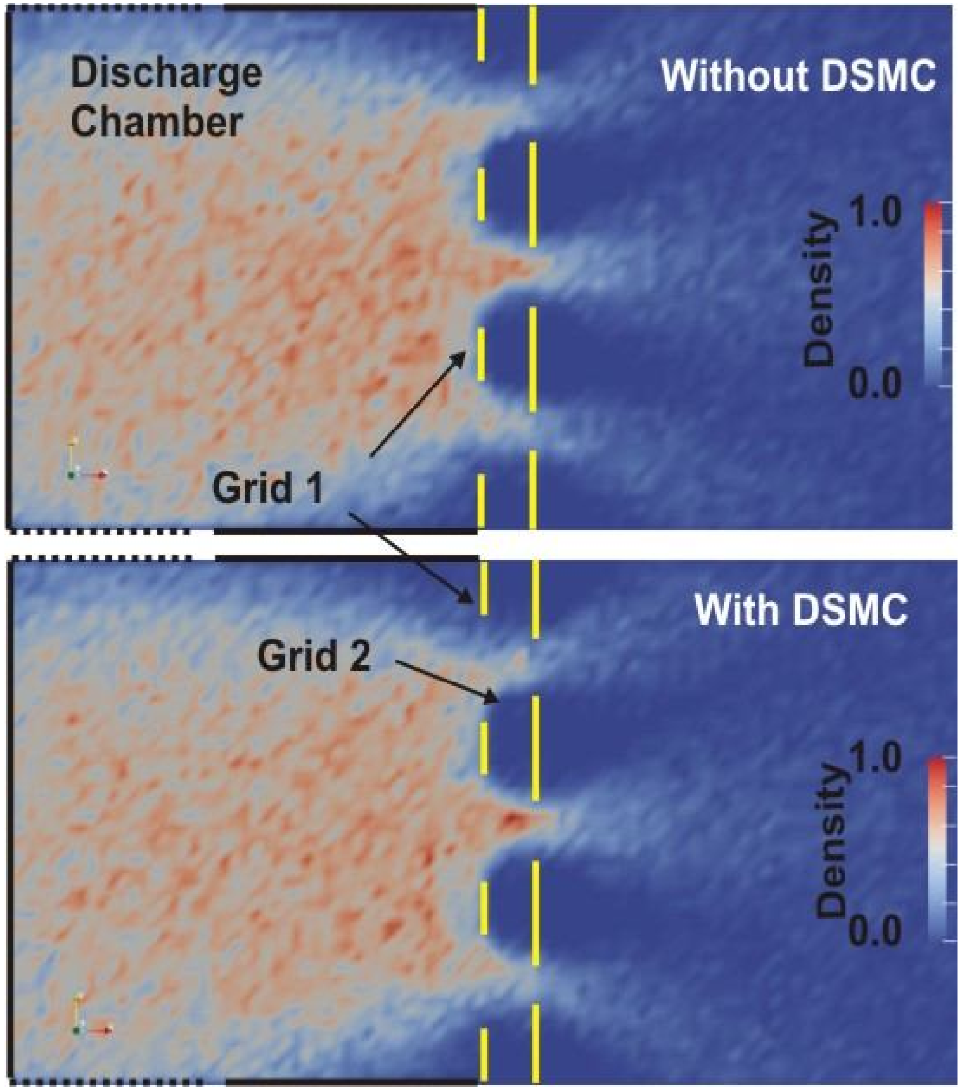
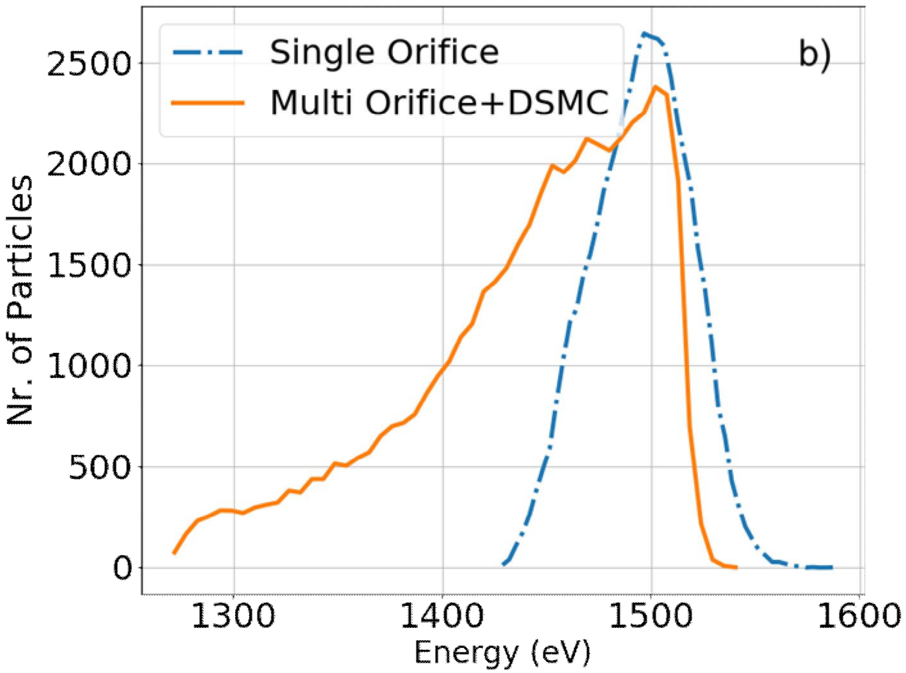
DSMC Simulations: Extraction system





Fluid / Hybrid modelling for Thruster

Levels of Plasma description

Level	Characteristic Parameters
Particle level	$f_{(\vec{r},\vec{v})}$
Fluid level	$\vec{v}_{\mathrm{d}(\vec{r})}, n_{(\vec{r})}$ und $T_{(\vec{r})}$
Global Modelling	n_0 and T_0

Fluid/Hybrid modelling

- A middle way out may be hybrid simulation of plasma to reduce computational efforts
- One of the specie typically electrons are considered fluid and ions as particle
- Comple fluid description involves MHD equations

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Fluid Model

- Due to collisions the system relaxes to Maxwellian distribution ie thermodynamic equilibrium
- In this state plasma can be described as fluid
- Fluid model derived from Vlasov equation are coupled to Maxwell's equations

Non equilibrium plasma flow model

$$Transient + Advective - Diffusive - Reactive = 0$$

Conservation of mass

$$\frac{\partial \rho_m}{\partial t} + \mathbf{u} \cdot \nabla \rho_m + \rho_m \nabla \cdot \mathbf{u} = 0$$

ρ_m = mass density, **u** = fluid bulk velocity

One can introduce source term

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{u} = \dot{n}_s$$

Which may further include Gain and Loss terms

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{u} = G_e - L_e$$

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Fluid Model

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Cauchy momentum equation

$$\rho \frac{d\mathbf{v}}{dt} = (\mathbf{J} \times \mathbf{B}) - \nabla p$$

Energy conservation equation

$$\rho_m \frac{\partial \epsilon}{\partial t} + \nabla \cdot [(\rho_m \epsilon + P)\,\mathbf{u}] = 0$$

Energy equation

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$$

Ohm's law

$$\mathbf{E} + (\mathbf{v} \times \mathbf{B}) = \eta \mathbf{J}$$

where Γ is ratio specific heat, σ is Plasma conductivity, and $\eta = \sigma^{-1}$

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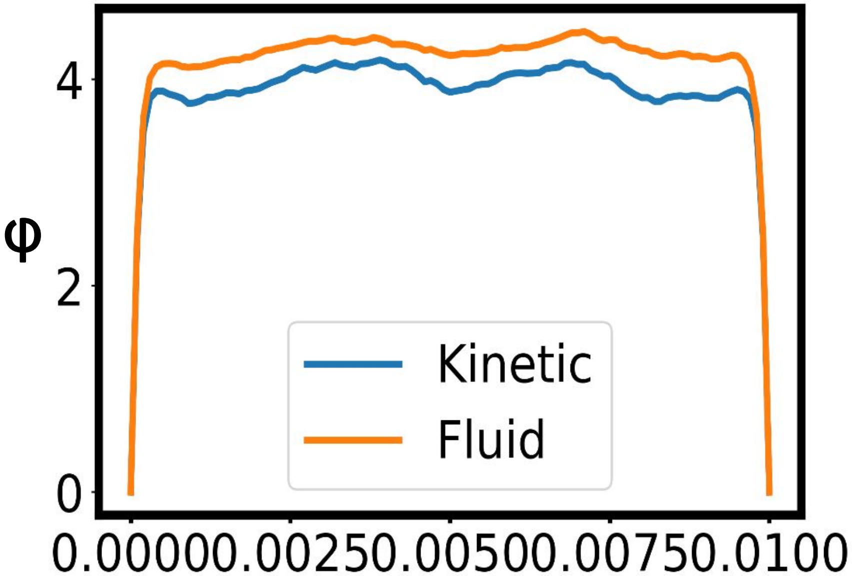
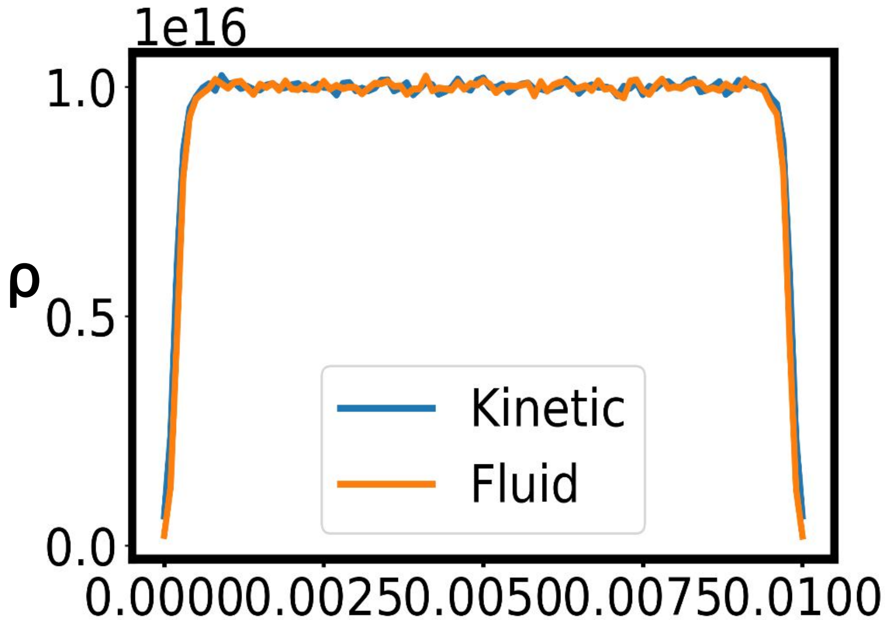
Fluid Model : Example plasma in box

Consider an example of neutral plasma in box.

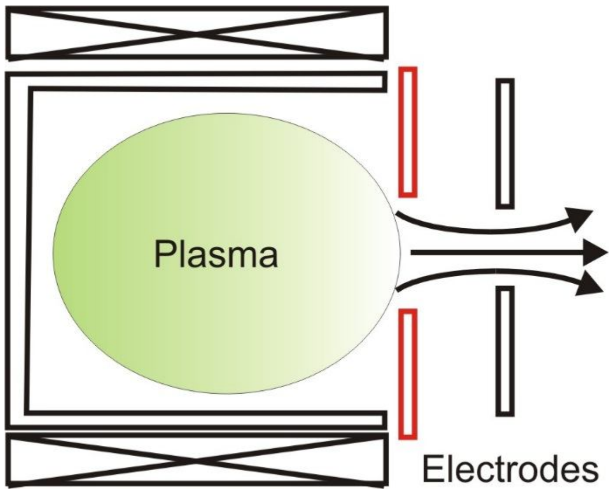
The box is hold at ground potential

Figure below compares potential and density distribution in 1D case



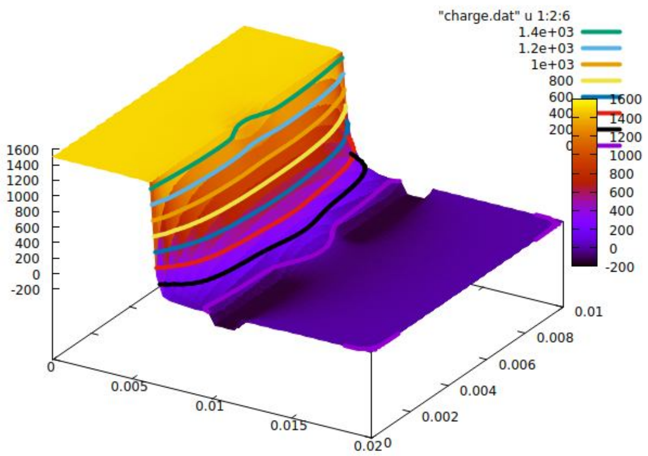


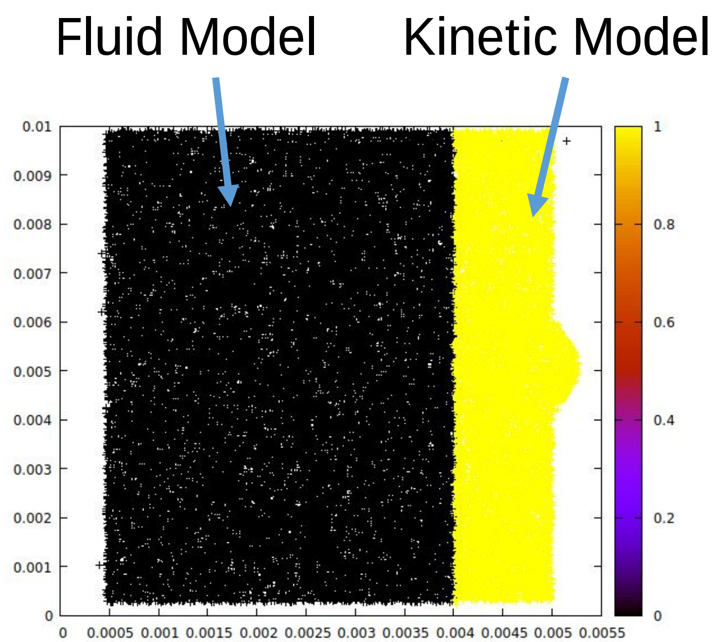
Fluid Model : 2D Extraction Hybrid model



Hybrid simulation:

1. Two species ions and electrons are separately described using Kinetic and fluid equation
2. The fluid and Kinetic properties are interchanged according to average velocities of the species





Global Modelling for Thruster

Levels of Plasma description

Level	Characteristic Parameters
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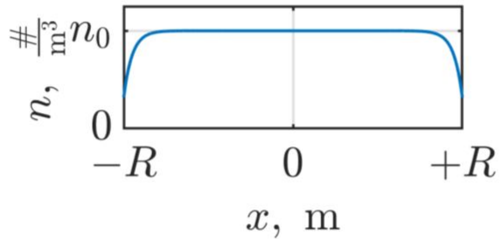
Fluid level	$\vec{v}_{\mathrm{d}(\vec{r})}, n_{(\vec{r})}$ und $T_{(\vec{r})}$
Global Modelling	n_0 and T_0

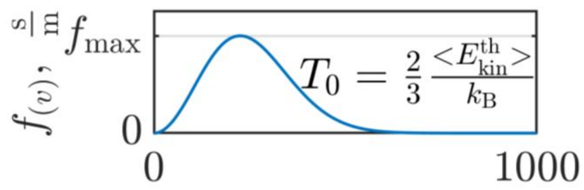
Global Modelling

- Estimation of density and temperature profile

Density profile

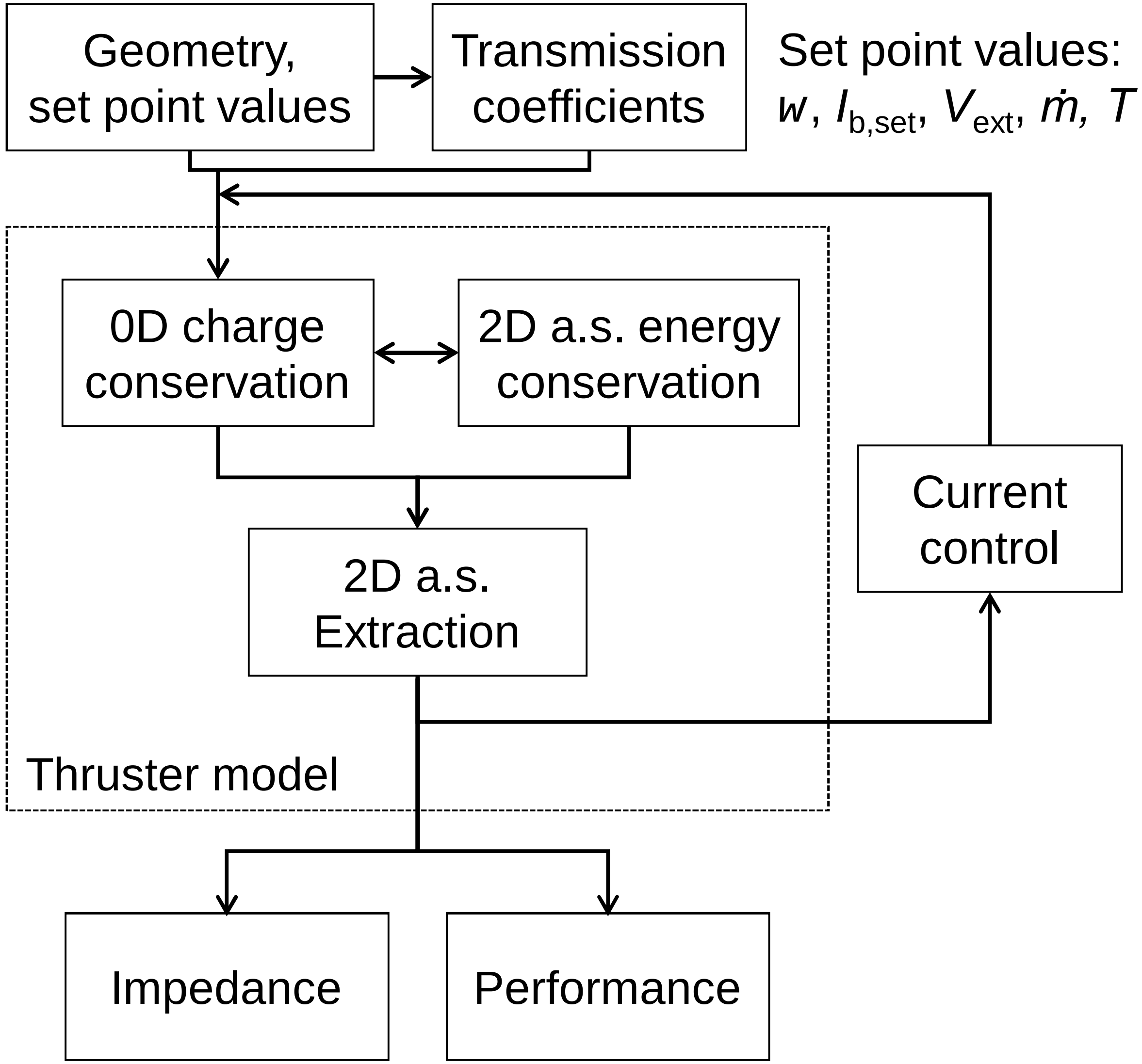


Velocity profile



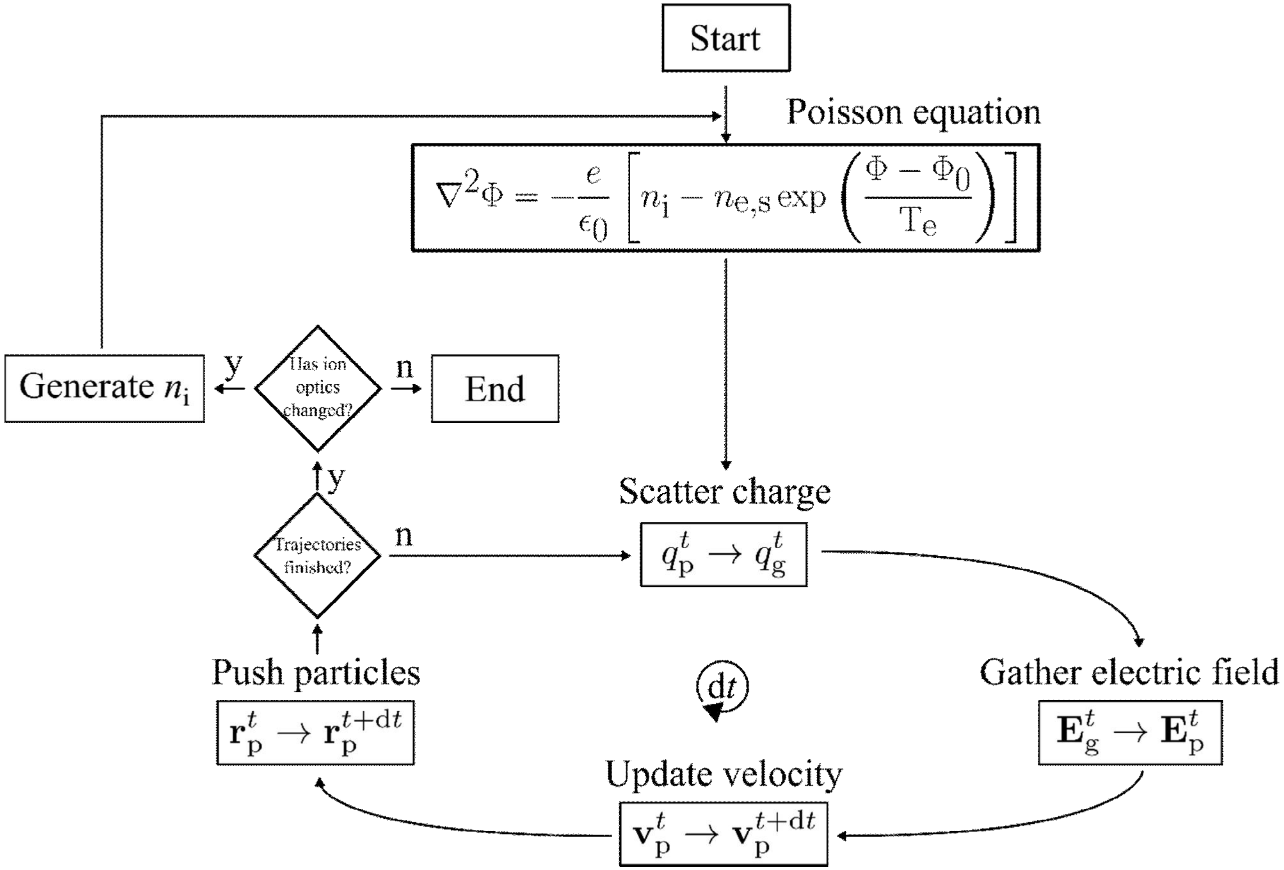
- System characterization using steady state condition
  - Energie
  - Teilchenanzahl

Ion Thruster : Global modelling



- Global modelling (0D) → neutral gas density n_0, ion density n_+, Temperature T
- Assuming Maxwell distribution and quasi neutrality, coupled to IGUN or IBSimu

2D Axi-symmetric modelling

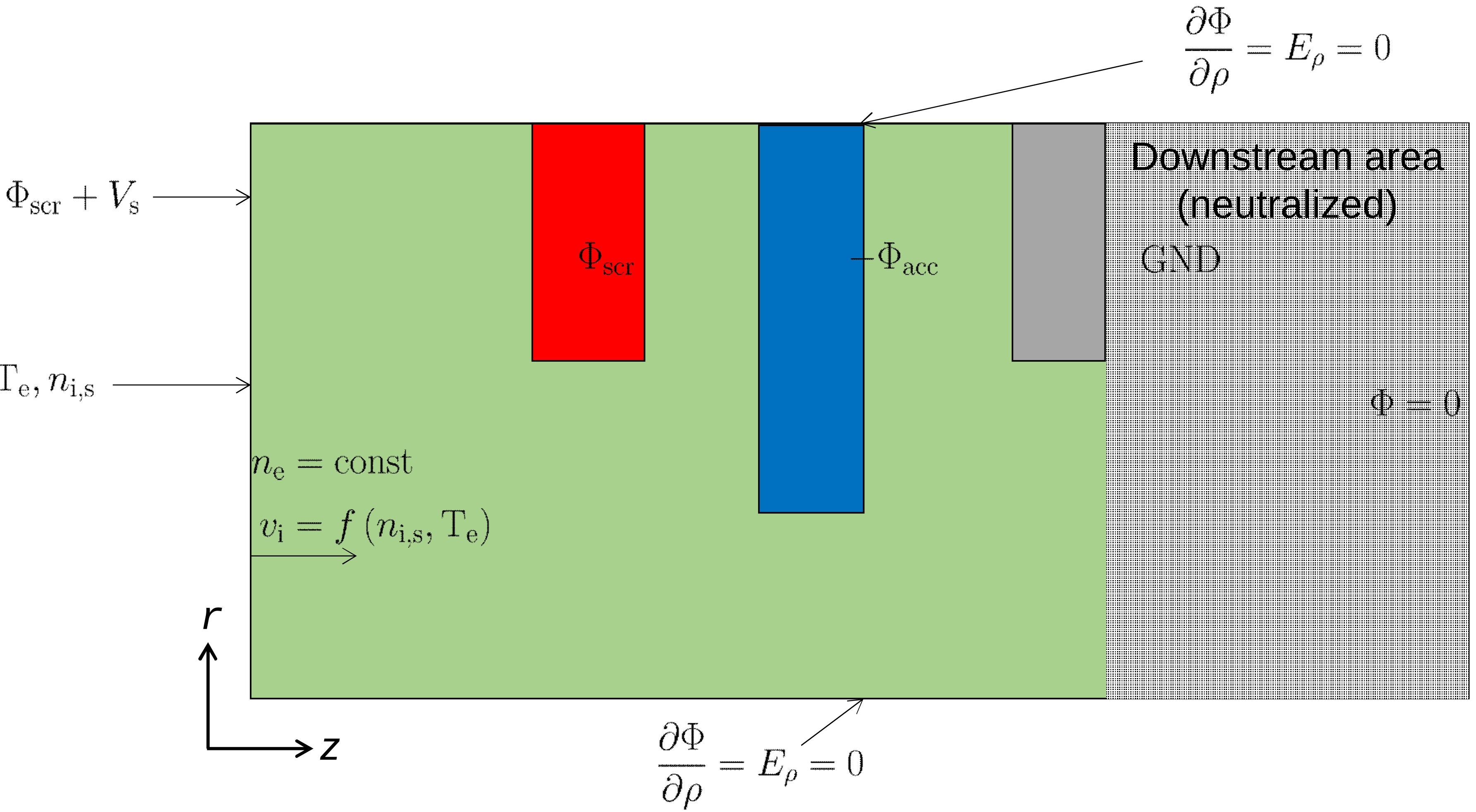


Modified 2D axi-symmetric Particle-In-Cell (PIC) method

- Full PIC: Vlasov-Poisson system → development of IEDF/EEDF
- Modified (here): particle trajectories, stationary IEDF

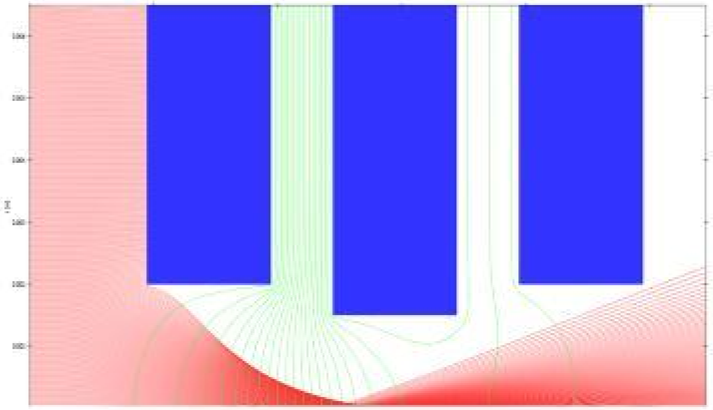
Ion Extraction

- Grid modelling: axi-symmetric model
- IBSimu and IGUN can calculate ion beam trajectories, emittance and optimization of grid geometries
- Extraction current is then coupled to control system feedback



Grid optimization

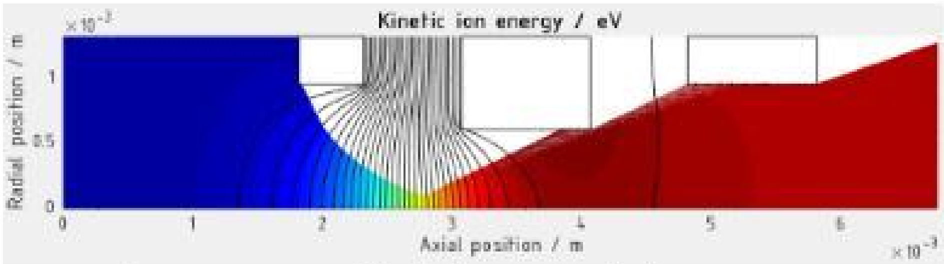
- Grid optimization in terms of Perveance of the ion beam



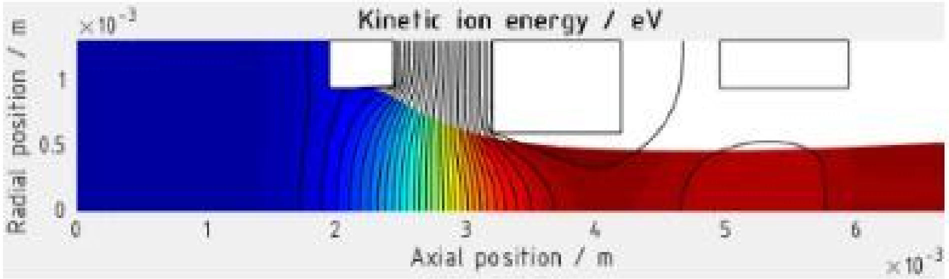
IBSimu Simulation

$$I = kV^{3/2}$$

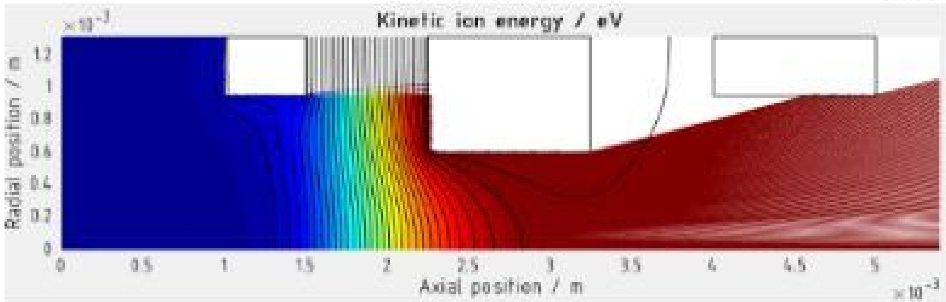
Low perveance



Optimal perveance

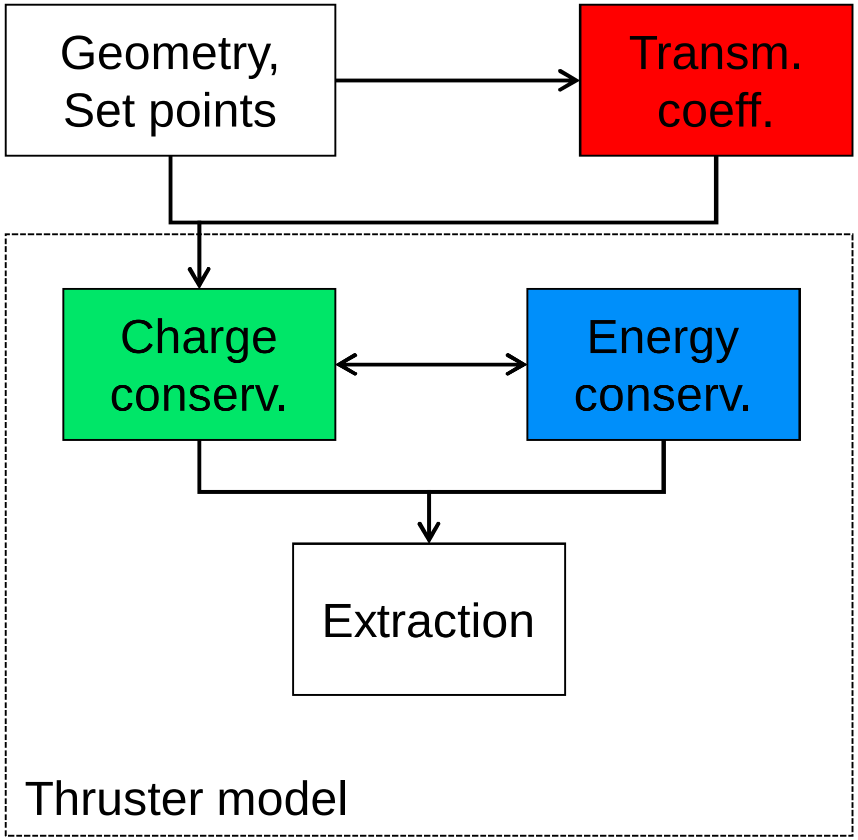


High perveance



Global modelling: Assumptions

- Global RF ion thruster model validated and verified for different thruster geometries and points of operation
- Fast simulation duration. Enables virtual prototyping
- Validated by measured RF current



- Temperature pre-assumed
- “Cold gas approximation” → ions not heated by RF electric field
- Maxwellian electrons
- Quasi-neutral plasma ($n_i = n_e$)
- Analytical treatment of plasma sheath ($n_i > n_e$) Not resolved in EM model
- No secondary electron emission
- Induction coil shows negligibly small axial pitch → axi-symmetry

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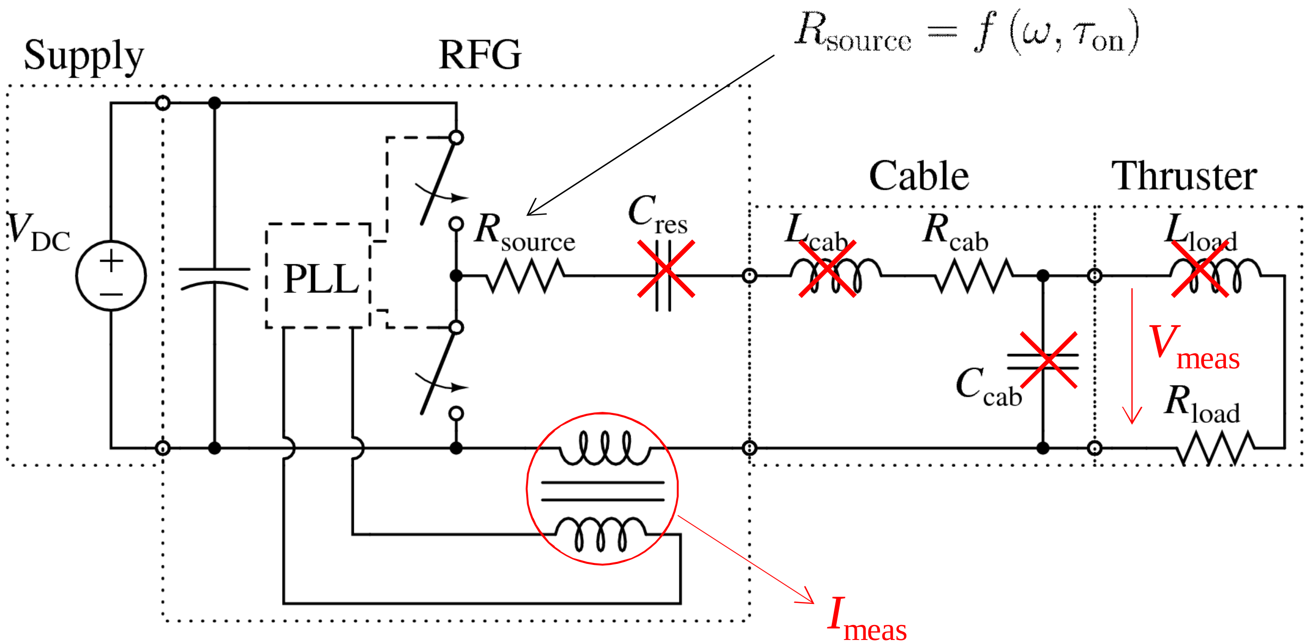
Global modelling: Peripherial components

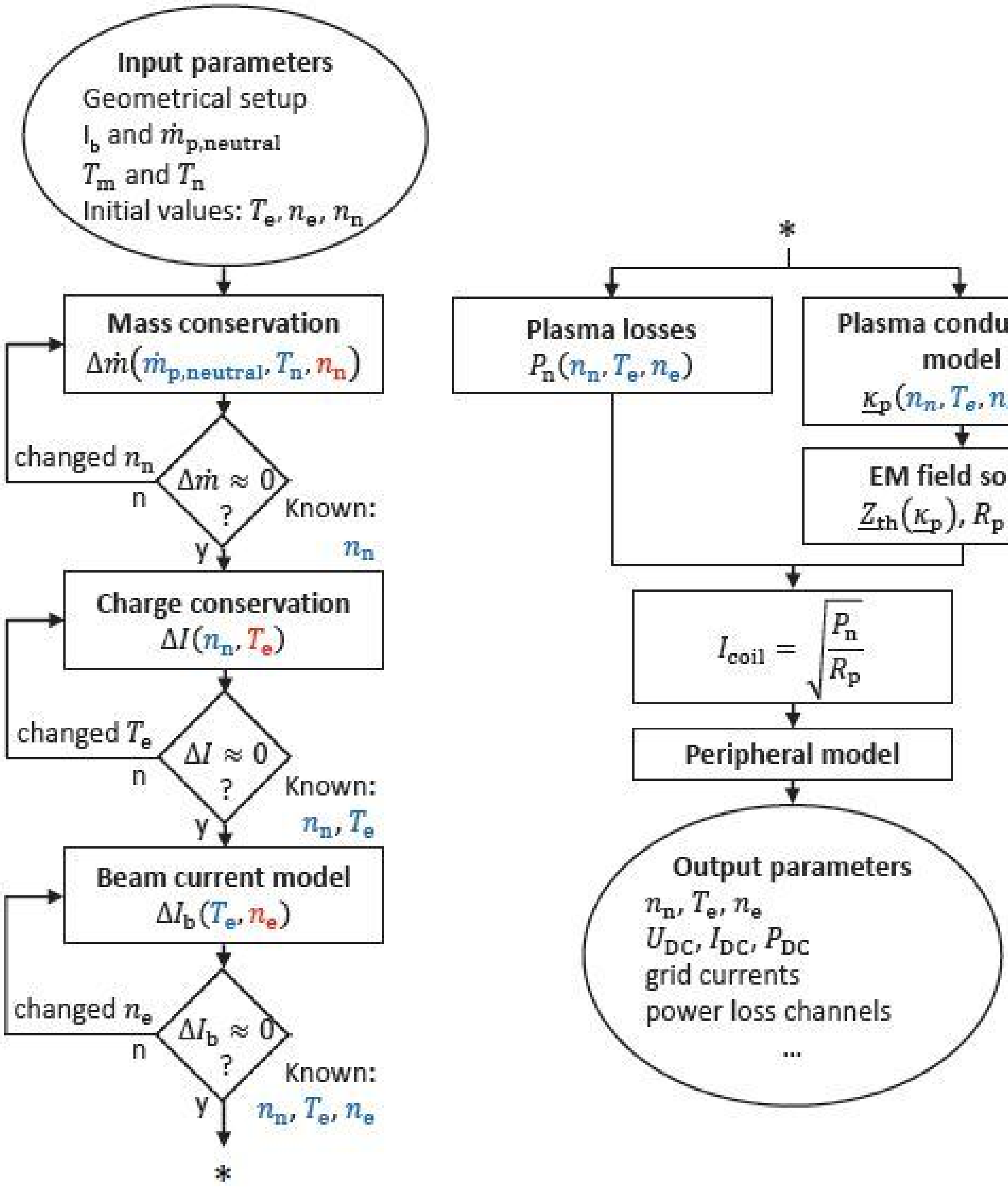


Power coupling in a RIT assembly

In resonance, reactive components cancel each other

Impedances bridged (rather than matched)
→ High efficiency possible

Global modelling: Flow-chart for conservation laws



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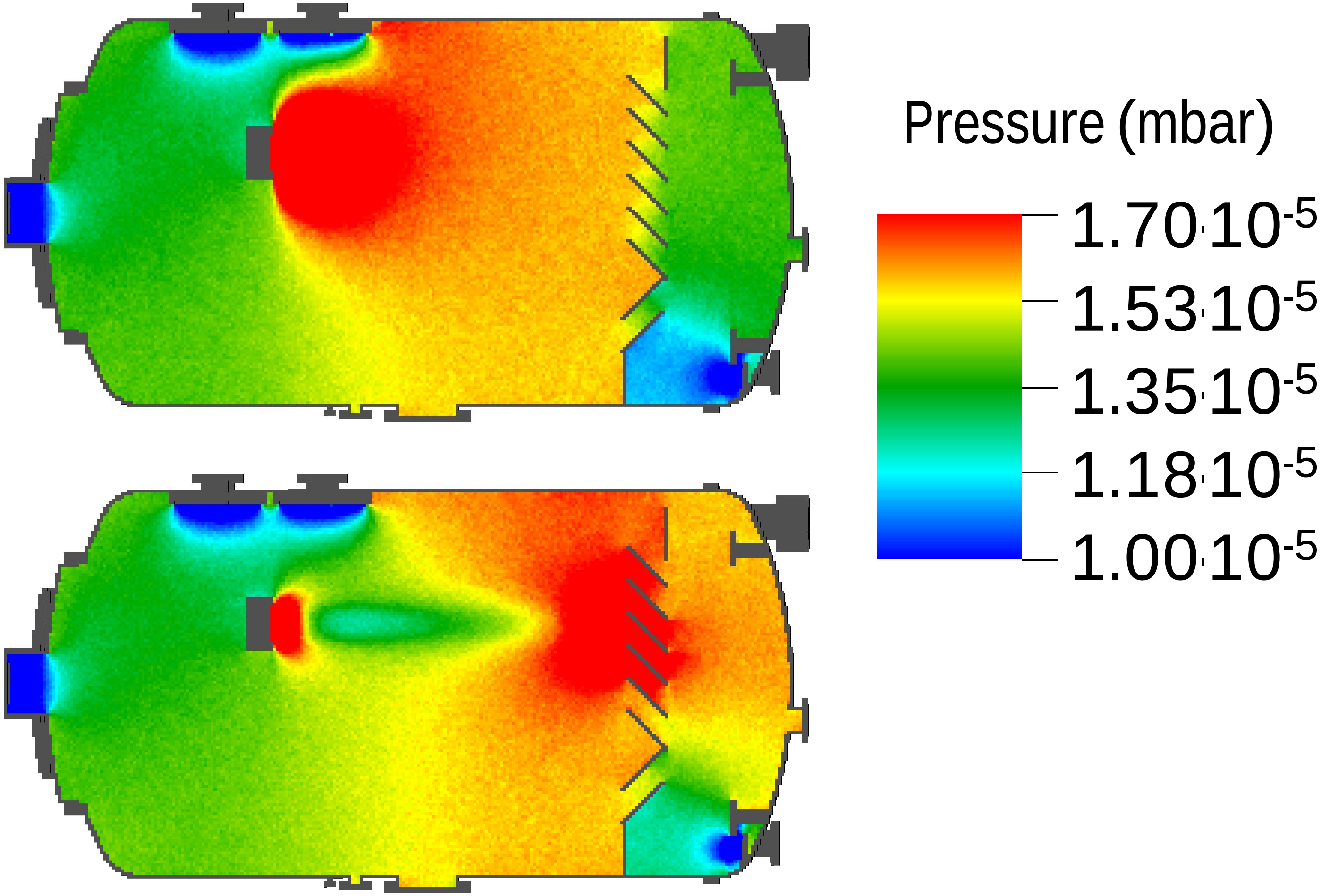
DSMC Simulations: Tank simulation

Without Extraction

with Extraction



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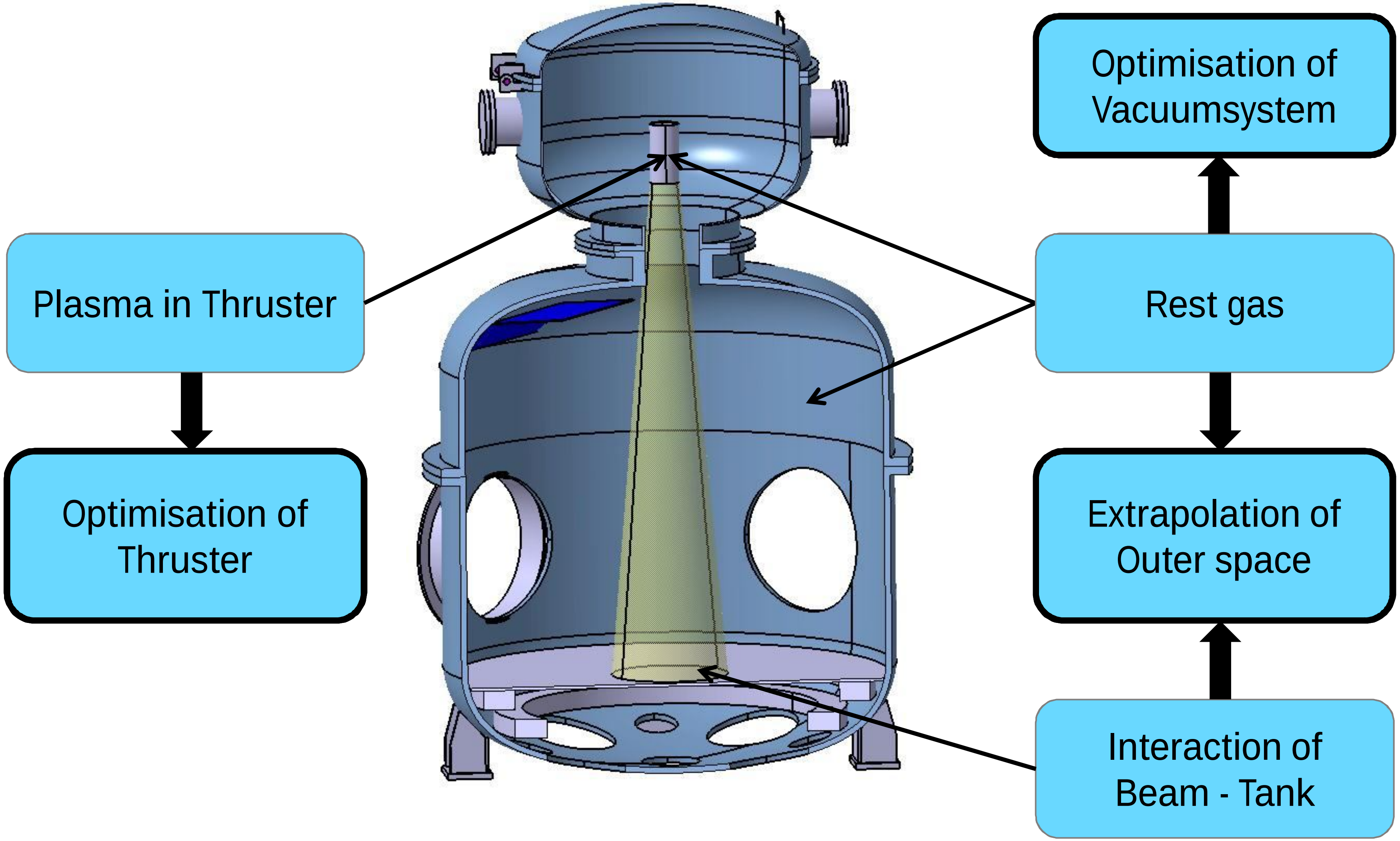
DSMC Simulations: Tank simulation



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